Mathematics Standards Clarification for Functions Conceptual Category

High School

2019

The Nevada Ready! Network

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Standards-Based Instruction for
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Contents

Interpreting Functions

Cluster

Understand the concept of a function and use function notation.

NVACS HSF.IF.A.1 (Major Supporting Work)

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students will make sense of problems when describing the relationship between domain and range when comparing graphs.
Instructional Strategies	When teaching this concept, it is helpful to relate the set form, graph, table, and verbal representations of function. • Students should experience a variety of types of situations modeled by function. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout future units and courses. • Provide representations of functions and non-functions. • Use proper vocabulary (domain, range, independent variable, dependent variable, function)
Prerequisite Skills	• Understand inputs and outputs
Connections Within and Beyond High School	• Use of function notation • Sequences and series • Different classes of functions throughout their mathematical education
Instructional Examples/Lessons/Tasks	• Blast Module (CCSD) • Interpreting Functions (shmoop)
Assessment Examples	• Functions & Function Notation (RPDP)

Interpreting Functions

Cluster

Understand the concept of a function and use function notation.

NVACS HSF.IF.A.2 (Major Supporting Work)

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Element	Exemplars
Standards for Mathematical Practice	● MP 4 Students use tables, graphs, and rules to represent relationships. ● MP 6: Students understand the meaning of symbols used in function notation.
Instructional Strategies	When teaching this concept, it is helpful to relate the set form, graph, table, and verbal representations of functions. ● Students should experience a variety of types of situations modeled by function. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout future units and courses. ● Give tasks that allow students to use function notation in different contexts. ● Ask students to evaluate functions using varying inputs. ● Question students about the meaning of function notation. ● Provide prompts that use a graph, table, or symbolic form to evaluate a function for a given input value.
Prerequisite Skills	● Evaluate expressions.
Connections Within and Beyond High School	● Working with all different types of equations and functions ● Transformations ● Modeling real-world situations
Instructional Examples/Lessons/Tasks	● Blast Module (CCSD)
Assessment Examples	● Functions and Function Notation Assessment (RPDP)

Interpreting Functions

Cluster

Understand the concept of a function and use function notation.

NVACS HSF.IF.A.3 (Major Supporting Work)

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by*

$$f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \text{ for } n \geq 1$$

Element	Exemplars
Standards for Mathematical Practice	• MP 2 Students demonstrate an understanding of quantities and their relationships. • MP 6 Students communicate precisely by using formulas recursively and explicitly.
Instructional Strategies	• Ask students to create explicit function rules for a sequence. • Ask students to create recursive rules for a sequence. • Ask students to compare and contrast both explicit and recursive forms of a function when possible.
Prerequisite Skills	• Identifying additive and multiplicative patterns in sets of numbers. • domain and range
Connections Within and Beyond High School	• Linear equations • Exponential equations • Arithmetic and geometric series
Instructional Examples/Lessons/Tasks	• Coherence Map (Achieve the Core)
Assessment Examples	• Functions and Function Notation Assessment (RPDP)

Interpreting Functions

Cluster

Interpret functions that arise in applications in terms of the context.

NVACS HSF.IF.B.4 (Major Supporting Work)

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*

Element	Exemplars
Standards for Mathematical Practice	• MP4 Students will model relationships of functions using contextual applications and graphical features.
Instructional Strategies	• Have students match graphs, tables and written functions together • Have students identify key features of graphs and relate them to written functions. • Have students connect graphs to descriptions of real events.
Prerequisite Skills	• Identify x- & y-intercepts from a graph or table.
Connections Within and Beyond High School	• Connections between $f(x)$, $f'(x)$, and $f''(x)$ • End behaviors
Instructional Examples/Lessons/Tasks	• Functions & Their Characteristics Matching Game (RPDP) • Graphing Stories (Desmos)
Assessment Examples	• Review of Functions (RPDP)

Interpreting Functions

Cluster

Interpret functions that arise in applications in terms of the context.

NVACS HSF.IF.B.5 (Major Supporting Work)

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *

Element	Exemplars
Standards for Mathematical Practice	• MP 2 Students connect a function to the context it represents using quantities.
Instructional Strategies	• Give students examples of functions and ask them to match the examples with the appropriate domain and range. • Compare discrete and continuous functions and describe the differences in terms of domain and range.
Prerequisite Skills	• Apply basic function concepts, e.g., by interpreting the features of a graph in the context of an applied problem.
Connections Within and Beyond High School	• Use function notation • Evaluate functions • Graph and interpret piecewise and step functions
Instructional Examples/Lessons/Tasks	• Oakland Coliseum (Illustrative Math)
Assessment Examples	• Sample Item 3278 (smarterbalanced.org) • Sample Item 3202 (smarterbalanced.org)

Interpreting Functions

Cluster

Interpret functions that arise in applications in terms of the context.

NVACS HSF.IF.B.6 (Major Supporting Work)

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *

Element	Exemplars
Standards for Mathematical Practice	• MP 5 Students use appropriate formulas or methods strategically based on the format of the question, i.e. given as a table, graph, equation, or a set of ordered pairs. • MP 6: Students understand the role that estimation plays with precision
Instructional Strategies	• Lecture and modeling examples • Scavenger hunts • WebQuests • Centers/stations
Prerequisite Skills	• Understand ratios and unit rates • Know how to calculate slope from a table, graph, equation, and ordered pairs • Understand slope is rate of change • Know the definition of an interval
Connections Within and Beyond High School	• Slope/average rate of change of a graph, table, equation, ordered pairs • Average rate of change for linear and non-linear equations • Determining whether an equation is increasing or decreasing over an interval • Interpret rate of change within the context of data • Derivatives

Instructional Examples/Lessons/Tasks	• HSF-IF.B.6 Tasks (Illustrative Math)
	• Average rate of change word problem: table (Khan Academy)
	• What Goes Up (Achieve the Core)
	• Group Activity
	1. Organize students into groups.
	2. Review the definition of slope connecting it with rate of change. Also, review the slope formula.
	3. Each group should be given three tables
	  • One representing a linear equation,
	  • One representing an exponential equation,
	  • One representing a quadratic equation
	4. Each group should receive problems to calculate the rate of change for a specified interval for each table.
	5. Have the students discuss the results with each other, and then the class.
	6. Discuss with the students the different vocabulary with the different types of equations.

Assessment Examples	• Lines and Slopes: ACT Math Geometry Review and Practice (prepscholar.com)

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.7 (Major Supporting Work)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

Element	Exemplars
Standards for Mathematical Practice	• MP1 Students make sense of problems by identifying the type of function given and persevere in solving them by determining what key features (values) are needed to graph by hand. • MP 5 Students use of the calculator to find key features in more complicated functions.
Instructional Strategies	• Start with graphing technology to have students determine general characteristics. • Match functions to graphs and graphs to functions.

Prerequisite Skills	• For linear functions
	  ◦ Find *x*- & *y*-intercepts (by hand)
	  ◦ Identify slope & *y*-intercept (by hand)
	  ◦ Identify slope & a point on the line (by hand)
	• For quadratic functions
	  ◦ Find zeros (by hand) by factoring and completing the square (simple)
	  ◦ Find axis of symmetry (by hand)
	  ◦ Find vertex (by hand)
	• Basic use of a graphing calculator

Connections Within and Beyond High School	• Transformations to move between parent function and given function
Instructional Examples/Lessons/Tasks	• Finding x-intercepts (RPDP) • Exploring Quadratics in Factored Form (RPDP) • Around the Vertex in 80 Days (RPDP) ◦ TI 84 Activity
Assessment Examples	• Intro. and Graphing Polynomials (RPDP) ◦ Problems #10 and #18

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.7a (Major Supporting Work)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

- Graph linear and quadratic functions and show intercepts, maxima, and minima.

Element	Exemplars
Standards for Mathematical Practice	• MP 5 Students use tools such as graphing technology to analyze graphs by verifying intercepts, maximum and minimum, after solving. • MP 7 Students make sure of structures in distinguishing between linear and quadratic functions in various forms.
Instructional Strategies	Linear Functions • Recall finding *x* and *y* intercepts Quadratic Functions • Recall that zeros are *x*-intercepts and constant term is the *y*-intercept. • Relate vertex form and leading coefficient to identifying the maximum or minimum. • Demonstrate that completing the square reveals the maximum or minimum when writing from standard form to vertex form.
Prerequisite Skills	• Factoring • Equation of Lines (slope intercept form, standard form, point slope form) • Quadratic Equations (standard form, vertex form)
Connections Within and Beyond High School	• Solving rational equations and stating end behavior • Vertical and horizontal asymptotes
Instructional Examples/Lessons/Tasks	• Using Quadratic Equations to Model Situations and Solve Problems (Illustrative Mathematics) • Finding Intersections (Illustrative Mathematics)
Assessment Examples	• PARCC sample assessment item HSF.IF.C.7A PARCC Sample Assessment Items 22. If $f(x) = 2x^2 - 8x + 9$, which statement regarding the vertex form of $f(x)$ is true? A. In vertex form, $f(x) = 2(x-2)^2 + 1$ and therefore has a minimum value of 1. B. In vertex form, $f(x) = 2(x-2)^2 + 1$ and therefore has a minimum value of −2. C. In vertex form, $f(x) = 2(x-2)^2 + 4.5$ and therefore has a minimum value of 4.5. D. In vertex form, $f(x) = 2(x-2)^2 + 4.5$ and therefore has a minimum value of −2. • PRACTICE Using Quadratic Equations to Model Situations and Solve Problems (Illustrative Mathematics) • Problem #2

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.7b (Major Supporting Work)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Element	Exemplars
Standards for Mathematical Practice	• MP 6 Students attend to precision by correctly sketching a graph of functions with given parameters. • MP 7 Students look closely for patterns in graphing functions and their transformations.
Instructional Strategies	Piecewise Functions • Graph all functions (entirely); identify domain of each; erase outside of domain. Absolute Functions • Define absolute value in terms of distance.
Prerequisite Skills	• Recognize, simplify, and solve square and cube roots. • Interpreting radical notation • Recognize square root, cube root, absolute value, and step functions. • Domain/Range
Connections Within and Beyond High School	• Average Rate of Change • Inverse Functions • Continuity, Discontinuity (removable, jump, infinite) • Precalculus Notes: Functions (RPDP)
Instructional Examples/Lessons/Tasks	• Polygraph: Piecewise Functions (Desmos) • MathLab Investigating Graphing Radical Functions (RPDP) • Graphing Radical Functions (RPDP) • Piecewise Function Matching Cards Activity (RPDP)
Assessment Examples	• Piecewise Functions (Illustrative Mathematics) • Absolute Value Functions (Part 1) (Illustrative Mathematics) • Absolute Value Functions (Part 2) (Illustrative Mathematics)

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.7c (Major Supporting Work)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

- Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students make sense of problems by determining the number of real or complex solutions. • MP 2 Students reason abstractly by analyzing parts of polynomial function in order to sketch the graph.
Instructional Strategies	• Review factoring quadratics, special cases (difference of squares, perfect square trinomial, sum and difference of cubes) • Understand effect of multiplicity of factors on the of shape of graph • Use zeros and multiplicities to construct rough sketches • Understand that a root or zero of a polynomial is a solution to the equation • Compare even and odd functions

Prerequisite Skills	• Factor polynomials
	• Identify
	  ◦ Degree
	  ◦ Leading term
	  ◦ Constant term
	  ◦ Relative maximum/minimum
	• Distinguish functions (constant, linear, quadratic, cubic, quartic, etc..)

Connections Within and Beyond High School	• State end behavior in limit notation • Graphs of rational functions with vertical and horizontal asymptote, extraneous solutions • Rewriting rational functions to reveal end behavior (polynomial division)
Instructional Examples/Lessons/Tasks	• End Behavior Part 1(Illustrative Mathematics) • End Behavior Part 2 (Illustrative Mathematics) • End Behavior of Rational Functions(Illustrative Mathematics)

Assessment Examples	• Rational Root Theorem and Finding Zeros (RPDP)
	• Intro. and Graphing Polynomials Part I Practice Test (RPDP)
	• How Many Solutions? (Illustrative Mathematics)
	• Essential Questions
	  ◦ What does a polynomial function look like?
	  ◦ How do I identify the zeros?
	  ◦ How do I identify the end behavior?

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.7d (Major Supporting Work)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

- (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students make sense of rational function in order to find the vertical and horizontal asymptote in order to sketch the graph. • MP 6 Students attend to precision by carefully factoring and identifying zeros.
Instructional Strategies	• Identify simple rational functions from graphs and equations. • Identify vertical and horizontal asymptotes. • Rewrite rational functions to reveal end behavior.
Prerequisite Skills	• Factoring and solving quadratics • Solving rational equations, common denominators • Polynomial division (synthetic division, long division, area model diagram/tabular method) • Remainder Theorem • End behavior and limit notation of even or odd functions • Domain and range
Connections Within and Beyond High School	• Solving rational equations • Precalculus/AP Calculus: limits and continuity, application to instantaneous rate of change and slope
Instructional Examples/Lessons/Tasks	• MathLab: Graphing Rational Functions (RPDP) • Graphs of Rational Functions Part 1 (Illustrative Mathematics) • Graphs of Rational Functions Part 2 (Illustrative Mathematics) • End Behavior of Rational Functions (Illustrative Mathematics)
Assessment Examples	• Rational Functions Practice Test RPDP,

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.7e (Major Supporting Work)

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students model graphs of exponential, logarithmic, and trigonometric functions with the use of transformations.
Instructional Strategies	• Graph parent functions and use transformations (color coded) to show the effects of all behaviors.
Prerequisite Skills	• Parent functions • Read/memorize values from unit circle • Tangent relationship with sine & cosine • Transformational shifts
Connections Within and Beyond High School	• Area under sine/cosine curves on an interval
Instructional Examples/Lessons/Tasks	• Exponential Graphing Stations Activity (RPDP) • MathLab: Graphing Periodic Functions (RPDP) • MathLab: Explore Transformations of Trig Functions (RPDP)
Assessment Examples	• Exponential & Logarithmic Functions Part 1 Practice Test • Problems #16 and #17

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.8a (Major Supporting Work)

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students make sense of problems by comparing different representations of functions, discovering features of families of functions, and finding key features. • MP 5 Students use appropriate tools to represent different functions to make sense of them. • MP 7 Students use the structure of a function to determine key features.
Instructional Strategies	• Review different types of factoring. • Give students a variety of functions and their graphs. Ask students to identify key features of both representations and describe similarities and differences of each. • Familiarize students with connecting zeros of a function with solving a quadratic by factoring. • Have students share the methods they have been taught in previous years. • Have students use different methods to show they get the same solutions.
Prerequisite Skills	• Factoring quadratic & polynomial functions • Completing the square • Solving equations • Simplify polynomial expressions • Graph different types of equations • Properties of exponents, including rational • Symmetry • Key feature of the graphs of linear, quadratic, radicals, logarithms, and other equations • Transformations
Connections Within and Beyond High School	• Relative & absolute Extrema • End behavior • Local maximum/minimums and points of inflection
Instructional Examples/Lessons/Tasks	• Building Quadratic Functions to Describe Situations (Part 3) (Illustrative Math) • Quadratic Formula Calculator and Solver (Math Warehouse) ◦ Interactive Parabolas Graphing Activity • Interpreting Functions F-IF.8a (shmoop)
Assessment Examples	• Interpreting Functions HS-IF.C.8a (Khan Academy)

Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.8b (Major Supporting Work)

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

Element	Exemplars
Standards for Mathematical Practice	• MP 6 Students attend to precision in writing equivalent expressions to represent situations that involve percent increase or decrease. • MP 8 Students look for repeating reasoning in classifying exponential growth or decay functions based on graph or equation.
Instructional Strategies	• Distinguish between growth rate (percent change, interest rate) and growth factor. • In the function, $a(1+r)^x$, the growth rate is r and growth factor is $1+r$ • Card Sort Activity ◦ Students work in pairs to match equations to verbal expressions in terms of interest calculated monthly, yearly, bimonthly, etc.
Prerequisite Skills	• Properties of exponents • Solving equations • Interpreting data representations (graph, table of values) • Translating verbal expressions to mathematical expressions • Graphing exponential functions
Connections Within and Beyond High School	• Solving exponential and logarithmic functions
Instructional Examples/Lessons/Tasks	• Exponential Functions and Percent Increase and Decrease (Desmos) • Expressed in Different Ways,(Illustrative Mathematics)
Assessment Examples	• Exponential Growth vs. Decay (study.com)

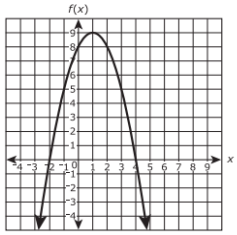
Interpreting Functions

Cluster

Analyze functions using different representations.

NVACS HSF.IF.C.9 (Major Supporting Work)

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students make sense of problems by comparing different representations of functions, discovering features of families of functions, and finding key features. • MP 5 Students use appropriate tools to represent different functions to compare properties. • MP 7 Students use the structure of a function to determine key features.
Instructional Strategies	• Give students two different functions in the form of an equation, graph, table or verbal description. Ask students to write a question comparing the functions. Share with other students. • Give students two quadratic functions- one as a parabola and one as a table. Ask the students to compare the key features. Which one travels farther? Which one reaches the greatest height?
Prerequisite Skills	• Function notation • Graph functions using a variety of tools • Compare properties of two linear functions • Rate of change • Zeros of functions
Connections Within and Beyond High School	• Find similarities between different families of functions
Instructional Examples/Lessons/Tasks	• Throwing Baseballs (Illustrative Mathematics)
Assessment Examples	• PARCC sample assessment item **10.** The figure shows a graph of the function of $f(x)$ in the xy-coordinate plane, with the vertex at $(1, 9)$ and the zeros at -2 and 4.  The function g is defined by $g(x) = -3x + 2$. Which statements are true? Select **all** that apply. **A.** $f(-2)$ is greater than $g(-2)$. **B.** $f(-1)$ is less than $g(-1)$. **C.** $f(0)$ is greater than $g(0)$. **D.** $f(1)$ is less than $g(1)$. **E.** $f(2)$ is greater than $g(2)$.

Building Functions

Cluster

Build a function that models a relationship between two quantities.

NVACS HSF.BF.A.1a (Major Supporting Work)

Write a function that describes a relationship between two quantities. *

- Determine an explicit expression, a recursive process, or steps for calculation from a context.

Element	Exemplars
Standards for Mathematical Practice	• MP3 Students will model the mathematics by writing functions to describe the relationship between two quantities. • MP7 Students look for and make use of structure by considering the differences between recursive and explicit expressions.
Instructional Strategies	• Have students explain how they found the next terms of a sequence and then work together to write in sequence and recursive form. The more the better. Explain that the formula is a standardized form for all explanations. • The most difficult part of this is the notation. Students are familiar with x and y, but have a difficult time understanding a_n and a_{n-1}. The more familiarity you can give your students with these new looking variables, the better. • Relate the x-y table with n and a_n.
Prerequisite Skills	• x-y table • Proportional reasoning • Describing what happens to the input value to obtain the output value
Connections Within and Beyond High School	• Writing functions from real world examples • Discrete and continuous functions • Direct/indirect relations • Exponential growth/decay rates/factors
Instructional Examples/Lessons/Tasks	• HSF-BF.A.1 Tasks (Illustrative Mathematics) • Creating Sequences (Open Middle) • Avi and Benita's Repair Shop (Desmos) • Algebra 1, Module 3, Topic A, Lesson 2 (engage NY)
Assessment Examples	• Modeling with Linear Functions Practice Test (RPDP) • Problems #13 , #14, #15, #16, #17, and #18

Building Functions

Cluster

Build a function that models a relationship between two quantities.

NVACS HSF.BF.A.1b (Major Supporting Work)

Write a function that describes a relationship between two quantities. *

- Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Element	Exemplars
Standards for Mathematical Practice	• MP4 Students will model building functions for multiple effects
Instructional Strategies	• Provide real world examples and have students write the formula to represent the situation.
Prerequisite Skills	• Understanding input/output values • *x-y* table
Connections Within and Beyond High School	• Going from concrete to abstract
Instructional Examples/Lessons/Tasks	• Game, Set, Flat (Desmos) • Exponential Parameters (Illustrative Mathematics) • Building Functions (Khan Academy)
Assessment Examples	• Exit Ticket (engage NY) • Modeling with Linear Functions Practice Test (RPDP) • Problems #13 , #14, #15, #16, #17, and #18

Building Functions

Cluster

Build a function that models a relationship between two quantities.

NVACS HSF.BF.A.1c (Major Supporting Work)

Write a function that describes a relationship between two quantities.*

- (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

Element	Exemplars
Standards for Mathematical Practice	• MP 7 Students will use the structure of functions to do compositions
Instructional Strategies	• Provide real world examples and have students write the formula to represent the situation. • Use graphing technology to allow students to test functions to see if they match the data.
Prerequisite Skills	• Understanding input/output values • *x-y* table
Connections Within and Beyond High School	• Going from concrete to abstract
Instructional Examples/Lessons/Tasks	• Building Functions (Khan Academy) • Precalculus and Advanced Topics Module 3, Topic B, Lesson 17: (engage NY) ◦ Great example on diving and atmospheric pressure
Assessment Examples	• Exit Ticket (engage NY) • Modeling with Linear Functions Practice Test (RPDP) ◦ Problems #13 , #14, #15, #16, #17, and #18

Building Functions

Cluster

Build a function that models a relationship between two quantities.

NVACS HSF.BF.A.2 (Major Supporting Work)

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students model with mathematics by looking at real world examples of arithmetic and geometric sequences. • MP 7 Students make use of patterns from arithmetic and geometric situations.
Instructional Strategies	• Technology to show patterns in exponential growth/decay • Real life examples of arithmetic and geometric sequences (population, growth/decay, money, radioactive decay, tiles and patterns, etc…) • Anchor Chart defining arithmetic and geometric sequences with color coding to show parts of equation in order to help with writing recursive and explicit forms
Prerequisite Skills	• Define common ratio/difference • Terms in a sequence • Function notation
Connections Within and Beyond High School	• Identifying vertical and horizontal asymptotes • State end behavior
Instructional Examples/Lessons/Tasks	• Desmos Exponential Growth and Decay (RPDP) • Activity uses Desmos • Exponential Growth vs Linear Growth I Task (Illustrative Mathematics) • Population and Food Supply Task (Illustrative Mathematics) • Snake on a Plane (Illustrative Mathematics) • Arithmetic VS Geometric (Open Middle)
Assessment Examples	• Functions and Function Notation Practice Test (RPDP) • Problem #16

Building Functions

Cluster

Build new functions from existing functions.

NVACS HSF.BF.B.3 (Major Supporting Work)

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Element	Exemplars
Standards for Mathematical Practice	• MP7 Students will make use of the structure of functions to draw conclusions. • MP8 Students will look for regularity in repeated reasoning to determine the effects of value changes in a function.

Instructional Strategies	• This standard should be taught with each parent function. Start with graphing, analyzing, and discussing characteristics of the parent function.
	• Question students
	  ◦ How to transform the graph and what that would do to the equation.
	  ◦ How does the transformation change the characteristics of the parent function? (domain, range, asymptotes, *x*- & *y*-intercepts, etc…)

Prerequisite Skills	• Creating *x*-*y* table in order to graph coordinate points. • Characteristics of a graph including domain & range
Connections Within and Beyond High School	• Graphing and analyzing functions
Instructional Examples/Lessons/Tasks	• Transformations of Functions 1 (TI Calculator Activities) • Transformations of Functions 2 (TI Calculator Activities) • Combining Transformations (TI Calculator Activities) • Changing Functions (TI Calculator Activities) • Function Transformations (RPDP) • Transformations Activities (Desmos) • HSF-BF.B.3 Tasks (Illustrative Mathematics)
Assessment Examples	• Review of Functions Practice Test (RPDP)

Building Functions

Cluster

Build new functions from existing functions.

NVACS HSF.BF.B.4a (Major Supporting Work)

Find inverse functions.

- (+) a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for $x \neq 1$.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students will model inverse functions with mathematics. • MP 5 Students use appropriate tools strategically, such as patty paper, coordinate plans, and tables to create inverse functions. • MP 7 Students will make use of the structure of functions to draw conclusions. • MP 8 Students will look for regularity in repeated reasoning to determine the effects of value changes in a function.
Instructional Strategies	• Pose questions that ask students to "undo" a function and help students connect to inverse operations (add/subtract, multiply/divide, square/square root). • Help students to discover that an ordered pair (x, y)on one function is the ordered pair (y, x) on its inverse and to graphically see that inverse functions are reflections over the line $y = x$. • Lecture and modeling examples • Stations/centers • WebQuests • Scavenger hunts • Patty paper activities
Prerequisite Skills	• Students understand the idea of an input resulting in an output. • Graphing basics • Function notation • Definitions of inverse, relation, and function • Inverse operations • Rearranging literals and formulas • Understand reflections • Domain and range
Connections Within and Beyond High School	• Students will not see the formal terminology of "inverse function" and the function notation until an Algebra II course. • Connecting algebra to geometry with reflections • Domain restrictions • Finding inverses of different types of equations • Composition of functions
Instructional Examples/Lessons/Tasks	• US Households (Illustrative Mathematics) • Temperatures in Degrees Fahrenheit and Celsius (Illustrative Mathematics) • Special Functions Notes (RPDP) • Patty Paper Activity (UTeach Ideas) • Inverse Functions (RPDP)
Assessment Examples	• Special Functions Practice Test (RPDP)

Building Functions

Cluster

Build new functions from existing functions.

NVACS HSF.BF.B.4b (Major Supporting Work)

Find inverse functions.

- (+) Verify by composition that one function is the inverse of another.

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students model with mathematics by checking if two functions in the real world are inverses. • MP 8 Students look for repeated reasoning to find the expressions for the corresponding inverse functions.
Instructional Strategies	• Students determine inverses of linear functions. • Verify algebraically whether or not two functions are inverses. • Students find a function and its inverse from data. • Using modeling with temperature to rearrange formulas, writing formula for the inverse function.
Prerequisite Skills	• Solving equations • Function notation • Input and output values
Connections Within and Beyond High School	• Solving rational equations • Extraneous solutions • Domain and range • End behavior • Vertical and horizontal asymptotes
Instructional Examples/Lessons/Tasks	• Inverse Functions (Illustrative Mathematics) • Shopping for Cookbooks (Illustrative Mathematics)
Assessment Examples	• PRACTICE Finding and Interpreting Inverse Function (Illustrative Mathematics)

Building Functions

Cluster

Build new functions from existing functions.

NVACS HSF.BF.B.4c (Major Supporting Work)

Find inverse functions.

- (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

Element	Exemplars
Standards for Mathematical Practice	• MP 2 Students reason abstractly and quantitatively by predicting values and writing equations to represent functions. • MP 8 Students look for repeated reasoning while interpreting various representations of data and comparing function and its inverse.
Instructional Strategies	• Understand the meaning of the inverse of a function. • Explain or show reasoning in determining whether functions are inverses of each other.
Prerequisite Skills	• Define function, inverse • Interpreting representations of data (table of values, list of values, graph) • Verbal descriptions • Function machines • Mapping notation
Connections Within and Beyond High School	• Vertible and non-invertible functions • Restrictions on domain
Instructional Examples/Lessons/Tasks	• Writing Inverse Functions to Solve Problems, (Illustrative Mathematics)
Assessment Examples	• PRACTICE Writing Inverse Functions to Solve Problems (Illustrative Mathematics)

Building Functions

Cluster

Build new functions from existing functions.

NVACS HSF.BF.B.4d (Major Supporting Work)

Find inverse functions.

- (+) Produce an invertible function from a non-invertible function by restricting the domain.

Element	Exemplars
Standards for Mathematical Practice	• MP6 Students will attend to the precision of domain restrictions
Instructional Strategies	• Give students graphs and tables of functions where finding an input for a given output is unclear because more than one-point share *y*-values.
Prerequisite Skills	• Interpreting graphs. • Reading tables.
Connections Within and Beyond High School	• Restrictions on domains will reappear when working with rational functions, logarithmic, and square root functions.
Instructional Examples/Lessons/Tasks	• Building Functions Questions and Skills (Khan Academy) • HSF-BF.B.4d Activities (Better Lesson) • Building Functions F-BF.4d (shmoop)
Assessment Examples	• Review of Functions Practice Test (RPDP) • Problems #12, #13, #14, #15, and #16

Building Functions

Cluster

Build new functions from existing functions.

NVACS HSF.BF.B.5 (Major Supporting Work)

Find inverse functions.

- (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Element	Exemplars
Standards for Mathematical Practice	• MP7 Students look for and make use of the structural connections between exponential and logarithmic functions.
Instructional Strategies	• Use examples that ask students to undo an exponential function where students cannot answer precisely using guess and check. • Give students opportunities to see the relationship between exponential and logarithmic functions with graphs and tables.
Prerequisite Skills	• Exponent properties • Input and output values • Transformations
Connections Within and Beyond High School	• It is extremely important for students to understand that logarithms find missing exponents.
Instructional Examples/Lessons/Tasks	• Exponentials and Logarithms I (Illustrative Mathematics) • Exponential Kiss (Illustrative Mathematics)
Assessment Examples	• Exponential & Log Functions Part II Practice Test (RPDP) • Problem #15

Linear, Quadratic & Exponential Models

Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

NVACS HSF.LE.A.1.a (Major Supporting Work)

Distinguish between situations that can be modeled with linear functions and with exponential functions.

- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Model the different functions using different representations. • MP 5 Students model with linear and exponential functions (population growth, making money, etc…). • MP 7 Look for differences or factors and make use of the structure to determine if a function is linear or exponential. • MP 8 Look for pattern in growth by equal differences or equal factors to determine if the function is linear or exponential.
Instructional Strategies	• Give students different situations and tables and ask students to find matches comparing features and family of functions. • Give students table and graphs to find rate of change.
Prerequisite Skills	• Rate of change • Linear functions • Exponential functions • Input/output table of values
Connections Within and Beyond High School	• Arithmetic sequences • Geometric sequences • Constant rate

Instructional Examples/Lessons/Tasks	• Comparing Linear and Exponentials Models (RPDP)
	  • Make a prediction about which model will produce the greatest amount of money
	• What Comes Next? (Desmos)
	  • Students make connections between graphs, tables, and equations of linear and exponential functions
	• Linear vs Exponential Card Sort (Desmos)
	  • Match equation to verbal translation, modeling with functions, look at table of values to determine if output represents.
	• US Population 1982-1988 (Illustrative Mathematics)
	• Basketball Bounces Assessment Variation 1 (Illustrative Mathematics)
	• Basketball Bounces Assessment Variation 2 (Illustrative Mathematics)
	• Finding Linear and Exponential Models (Illustrative Mathematics)

Assessment Examples	• Exponential & Log Functions Part 1 Practice Test (RPDP) • Problems #1 through #14 • Predicting Populations (Illustrative Mathematics)

Linear, Quadratic & Exponential Models

Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

NVACS HSF.LE.A.1B (Major Supporting Work)

Distinguish between situations that can be modeled with linear functions and with exponential functions.

- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Element	Exemplars
Standards for Mathematical Practice	• MP 2 Students use repeated reasoning to recognize linear functions. • MP 4 Students model the different functions using different linear situations. • MP 7 Students use structure to recognize linear functions. • MP 8 Students look for patterns in which the rate of change involves adding a constant for equal units of change in the input.

Instructional Strategies	• Provide students with a linear situation.
	  • Ask students to recognize the constant rates of change and create a linear model.
	• Give students cards using multiple representations.
	  • Ask students to discuss rate of change with classmates and match the cards.

Prerequisite Skills	• Slope • Rate of change • Input and output • Linear function
Connections Within and Beyond High School	• Arithmetic sequences • Geometric sequences
Instructional Examples/Lessons/Tasks	• US Population 1982-1988 (Illustrative Mathematics)
Assessment Examples	• Modeling with Linear Functions Practice Test (RPDP) • Problems #10 and #11

Linear, Quadratic & Exponential Models

Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

NVACS HSF.LE.A.1C (Major Supporting Work)

Distinguish between situations that can be modeled with linear functions and with exponential functions.

- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Element	Exemplars
Standards for Mathematical Practice	• MP 2 Students use quantitative reasoning to recognize exponential functions. • MP 4 Students model the different functions using different exponential situations. • MP 7 Students use structure to recognize exponential functions. • MP 8 Students look for a pattern in which the quantity grows or decays by a constant percent rate per unit interval relative to another.

Instructional Strategies	• Provide students with an exponential situation.
	  • Ask students to recognize if the quantity grows or decays.
	• Give students cards using multiple representations.
	  • Ask students to discuss rate of change per unit, shape of graph, and function behavior with classmates and match the cards.

Prerequisite Skills	• Slope • Rate of change • Input and output • Linear function
Connections Within and Beyond High School	• Arithmetic sequences • Geometric sequences
Instructional Examples/Lessons/Tasks	• MathLab Modeling Cancer Cells with M&Ms (RPDP) • Basketball Bounces, Assessment Variation 1 (Illustrative Mathematics) • Basketball Bounces, Assessment Variation 2 (Illustrative Mathematics)
Assessment Examples	• Exponential and Log Functions Part I Practice Test (RPDP) • Problems #9 through #12

Linear, Quadratic & Exponential Models

Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

NVACS HSF.LE.A.2 (Major Supporting Work)

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students model with mathematics by determining if situation represents an arithmetic or geometric sequence. • MP 7 Students make use of structure of functions in order to sketch a graph and interpret data from graph/table to determine *n*th terms.

Instructional Strategies	• Differentiating between arithmetic and geometric sequences
	• Closed form, recursive form
	• Common ratio versus common difference
	• Zero term versus first term
	• Real world problems
	  ◦ Growth rate
	  ◦ Population
	• Using spreadsheets to graph functions

Prerequisite Skills	• Writing equations • Function notation
Connections Within and Beyond High School	• End behavior • Polynomial functions to model data
Instructional Examples/Lessons/Tasks	• Towering Sequence (Hanoi) (GeoGebra) • Moving Checkers (GeoGebra) • Introducing Geometric Sequences (Illustrative Mathematics) • Using Technology to Work with Sequences (Illustrative Mathematics) • Situations and Sequence Types (Illustrative Mathematics)

Assessment Examples	• Functions and Function Notation Practice Test (RPDP)
	  ◦ Problems #10 and #11
	• Quadratic Functions Practice Test (RPDP)
	  ◦ Problem #19

Linear, Quadratic & Exponential Models

Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

NVACS HSF.LE.A.3 (Major Supporting Work)

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Element	Exemplars
Standards for Mathematical Practice	• MP 7 Students will use the structural aspects of different types of function graphs to determine which they are seeing
Instructional Strategies	• Review graphs of all of the types of functions you have covered this year. • Have students identify key characteristics of the types of graphs. • Give students a series of unidentified graphs and have them determine the types and match them to available functions.
Prerequisite Skills	• Graphing of various functions
Connections Within and Beyond High School	• This connects to all of the functions that have been covered this year.

Instructional Examples/Lessons/Tasks	• Tying Knots (RPDP)
	  ◦ Linear
	• Modeling with Polynomial Functions (RPDP)
	  ◦ Polynomials
	• Modeling with Exp and Log Functions (RPDP)
	  ◦ Exponential

Assessment Examples	• Exponential and Log Functions Part I (RPDP) ◦ Problem #23

Linear, Quadratic & Exponential Models

Cluster

Construct and compare linear, quadratic, and exponential models and solve problems.

NVACS HSF.LE.A.4 (Major Supporting Work)

For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Element	Exemplars
Standards for Mathematical Practice	• MP 7 Students will use the structural aspects of different types of function graphs to determine which they are seeing.
Instructional Strategies	• Look at a modeling situation that will not fit into a linear, quadratic and polynomial model. • Review shapes of parent functions so the students will see that the data best fits an exponential model.
Prerequisite Skills	• Knowledge of exponential and logarithmic functions
Connections Within and Beyond High School	• This connects to the parent functions that students have learned in early parts of the class
Instructional Examples/Lessons/Tasks	• Modeling with Exp and Log Functions (RPDP)
Assessment Examples	• Exponential and Log Functions Part I (RPDP) • Problem #23 and 24

Linear, Quadratic & Exponential Models

Cluster

Interpret expressions for functions in terms of the situation they model.

NVACS HSF.LE.B.5 (Major Supporting Work)

Interpret the parameters in a linear or exponential function in terms of a context.

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students make sense and persevere in interpreting, evaluating and constructing linear and exponential functions. • MP 2 Students reason quantitatively by solving real world problems (radioactive decay of plant fossils) and paying close attention to appropriate domain and range.
Instructional Strategies	• Students understand the meanings of a, b, and the exponent in the equation $y=ab^x$. • Students use function notation to represent linear or exponential relationships. • Use technology to verify a reasonable graphing window for an exponential function. • Interpret and provide justification in explaining why data represents linear, exponential functions.
Prerequisite Skills	• Writing expression to represent situation • Evaluating expressions • Using a table of values • Reasoning backwards • Domain and range
Connections Within and Beyond High School	• End behavior • Derivative of functions
Instructional Examples/Lessons/Tasks	• Interpreting and Using Exponential Functions (Illustrative Mathematics) • Predicting Populations (Illustrative Mathematics)
Assessment Examples	• PRACTICE Interpreting and Using Exponential Functions (Illustrative Mathematics)

Trigonometric Functions

Cluster

Extend the domain of trigonometric functions using the unit circle.

NVACS HSF.TF.A.1 (Major Supporting Work)

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Element	Exemplars
Standards for Mathematical Practice	● MP 7 By relating the radius to radians students will build an understanding of the relation between arc length and angle measures. ● MP 8 Students will understand that circles with different radii will not change the radian measure of an angle.
Instructional Strategies	● Create circles with various radii and use string to approximate one radian on each. Compare the different circles with one radian marked to see how they relate. ● Have students explore the relationship between radians and the circumference of a circle.
Prerequisite Skills	● Circumference of a circle ● Rotations ● Angles in standard position
Connections Within and Beyond High School	● Graphing trigonometric functions ● Radians are necessary for forming the unit circle ● Advance math courses tend to use radian measures more often than degree measures.
Instructional Examples/Lessons/Tasks	● Trigonometry and Angles Notes Part I (RPDP) ● Trigonometry and Angles Notes Part II (RPDP)
Assessment Examples	● Trigonometry and Angles Part I Practice Test (RPDP) ● Trigonometry and Angles Part II Practice Test (RPDP)

Trigonometric Functions

Cluster

Extend the domain of trigonometric functions using the unit circle.

NVACS HSF.TF.A.2 (Major Supporting Work)

Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Element	Exemplars
Standards for Mathematical Practice	• MP 1 Students need to be able to explain the relationship between coterminal angles and reference angles. • MP 7 Students will be able to explain the structure of and patterns that exist within the unit circle.
Instructional Strategies	• Relate angles beyond 360 degrees or 2π radians to things like snowboarding with 720s and 1440s. These make nice quick videos as well. • Develop specific angles on a circle to include 0^o,30^o, 45^o,60^o, 90^o,120^o, 135^o,150^o, 180^o,210^o, 225^o,240^o, 270^o,300^o, 315^o,330^o, 360^o along with their radian equivalents (create these in different groups: the multiples of 60, the multiples of 30 and the multiples of 45) • Students will create a chart to relate angles greater than 360^o or 2π to the angles marked on the circle
Prerequisite Skills	• Understand reference angles • Convert between radian and degree measures
Connections Within and Beyond High School	• Angles used within this lesson will be used to build the unit circle. • Coterminal angles • Graphing trigonometric functions • X games (terminology of the rotations for snowboarding, skateboarding, etc…)
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP) • Algebra II, Module 2, Topic A, Lesson 5 (engage NY)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP)

Trigonometric Functions

Cluster

Extend the domain of trigonometric functions using the unit circle.

NVACS HSF.TF.A.3 (Major Supporting Work)

(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Element	Exemplars
Standards for Mathematical Practice	• MP 2 Students need to be able to relate the values of special triangles to the points on the unit circle. • MP 5 Students will understand that the unit circle is a tool for determining the trigonometric function values of special angles. • MP 8 Students will understand that calculating the values of trigonometric function values in quadrant I can be used to determine the values in all other quadrants.

Instructional Strategies	• Draw a 45-45-90 special right triangle in the first quadrant.
	  ◦ The adjacent side is the *x*-value.
	  ◦ The opposite side is the *y*-value.
	  ◦ With a hypotenuse of 1, the *x*-value is the cosine of the angle, and the *y*-value is the sine of the angle.
	  ◦ Repeat with 30-60-90 special right triangle and 60-30-90.
	• Trigonometry! Simple Hand Trick for Memorizing Values (YouTube)

Prerequisite Skills	• Students need to know the relationships in special right triangles (30-60-90 and 45-45-90). • Students need to recall the rules for reflections across the *x* and *y* axis. • Students need to remember the signs of both the *x* and *y* coordinates in each of the four quadrants of the coordinate plane
Connections Within and Beyond High School	• Exact unit circle values will be used for many problems throughout the trigonometry unit. • Graphing trigonometric functions
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP) • Algebra II, Module 2, Topic A, Lesson 4 (engage NY) • Algebra II, Module 2, Topic A, Lesson 5 (engage NY) • HSF-TF.A.3 Tasks (Illustrative Mathematics)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP)

Trigonometric Functions

Cluster

Extend the domain of trigonometric functions using the unit circle.

NVACS HSF.TF. A.4 (Major Supporting Work)

(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Element	Exemplars
Standards for Mathematical Practice	• MP 5 Use the unit circle as a tool to show the periodic nature of trigonometric functions. • MP 7 Use the structure of the unit circle to show the periodic nature of trigonometric functions.
Instructional Strategies	• Ask students about the relationships of trigonometric values for angle pairs such as $\frac{\pi}{3}$ and $-\frac{\square}{3}$ • Have students explain the relationship between the graphs of an angle and its negative such as π and $-\square$. • Have students investigate which trigonometric functions have even and odd symmetry using the unit circle. • Connect with Physics teacher for possible cross-curricular teaching.
Prerequisite Skills	• Knowledge of the trigonometric values from the unit circle
Connections Within and Beyond High School	• Connections to polynomial functions being odd or even. • Cross-curricular connections to sound waves and light waves.
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP) • Trigonometric Identities (Texas Instruments) • Algebra II Module 2, Topic A, Lesson 10 (engage NY)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP)

Trigonometric Functions

Cluster

Model periodic phenomena with trigonometric functions.

NVACS HSF.TF. B.5 (Major Supporting Work)

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students will be able to model real world situations with sinusoidal functions.
Instructional Strategies	• Have students use real world situations like ferris wheels, car tires, ocean waves to generate data. Then use the data to build models using the sine or cosine functions.
Prerequisite Skills	• Understanding radian measure • Graphing trigonometric functions • Definitions of amplitude, frequency, and midline of trigonometric functions
Connections Within and Beyond High School	• Using the regression function of a technology tool is a method that applies to any type of functions. • Many professions use periodic functions to model real world phenomena. For example, heart beats can be modeled using the sine function.
Instructional Examples/Lessons/Tasks	• Graphing the Sine Function using Amplitude, Period, and Vertical Translations (Desmos) • Ferris Wheel 3 Act Task (Dan Meyer) • Algebra II, Module 2, Topic A, Lesson 2 (engage NY) • Graphs of Sine, Cosine, and Tangent (Texas Instruments) • HSF-TF.B.5 Tasks (Illustrative Mathematics)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP)

Trigonometric Functions

Cluster

Model periodic phenomena with trigonometric functions.

NVACS HSF.TF.B.6 (Major Supporting Work)

(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

Element	Exemplars
Standards for Mathematical Practice	• MP 3 Students will construct a viable argument about why restricting the domain of a function allows it to have an inverse that is also a function.
Instructional Strategies	• Use visual representations to show the graph of a function and its reflection over y = x and determine if it is a function. Start with linear functions and progress through quadratic and end with trigonometric functions.
Prerequisite Skills	• Students need to understand the meaning of an inverse function. • Students need to understand how to graph trigonometric functions. • Students need to understand the domain and range of trigonometric functions.
Connections Within and Beyond High School	• Using inverse functions to solve trigonometric equations • Inverse relations for other functions such as linear and quadratic • Restricting the domain is a common method for solving many advanced mathematics problems.
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP) • Sample Assignments (shmoop)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP) • Sample Assignments (shmoop)

Trigonometric Functions

Cluster

Model periodic phenomena with trigonometric functions.

NVACS HSF.TF.B.7 (Major Supporting Work)

(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4 Students will apply inverses to right triangle trigonometry in order to solve for an angle in modeling situations. • MP 6 Students will understand when to use an exact vs. approximate answer when solving a trigonometric equation based on the context of the problem.
Instructional Strategies	• Provide contextual problems for which finding the solution requires finding an angle. • Make connections from a single angle solution to an infinite number of solutions using the unit circle, when applicable in context. For example, riding a ferris wheel. • Have students draw the diagram and identify the given information to determine which trigonometry function (for right triangles) or law (for oblique triangles) is appropriate for the situation.
Prerequisite Skills	• Students need to understand the meaning of an inverse function. • Students need to understand the right triangle trigonometry ratios (sine, cosine, tangent). • Knowledge of the unit circle
Connections Within and Beyond High School	• Used to solve for angles of right triangles and when using the law of sines and the law of cosines for oblique triangles. • Used in many real-world applications involving distances and direction
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP) • Sample Asignments (shmoop)

Trigonometric Functions

Cluster

Prove and apply trigonometric identities.

NVACS HSF.TF.C.8 (Major Supporting Work)

Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Element	Exemplars
Standards for Mathematical Practice	• MP 7 Students will discover the Pythagorean identity by looking at patterns and then verify it by looking at right triangles.

Instructional Strategies	• Create a table with angles identified.
	  ◦ Have students find the sine and cosine for each value.
	  ◦ Have students square the sine and cosine values and add them together.
	  ◦ Students should discover the relationship from these patterns.

Prerequisite Skills	• Pythagorean Theorem • Sign value of (x,y) in each of the quadrants • Relationships between sine, cosine and tangent
Connections Within and Beyond High School	• Pythagorean identity can be used in generating all 6 trigonometry values when given only one to start. • The Pythagorean identity is used as part of verifying many other trigonometric identities. • The Pythagorean identity allows many calculus problems which involve trigonometry functions to be manipulated into a recognizable form.
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP) • Trigonometric Identities (Texas Instruments • Algebra II, Module 2, Topic B, Lesson 15 (engage NY)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP)

Trigonometric Functions

Cluster

Prove and apply trigonometric identities.

NVACS HSF.TF.C.9 (Major Supporting Work)

(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Element	Exemplars
Standards for Mathematical Practice	• MP 5 Students will use the appropriate identity (tool) for the given situation.
Instructional Strategies	• The proof portion of this standard is reserved for advanced math courses such as a Pre-Calculus course. Using the formulas is appropriate for regular level courses. • Use specific instances of angle addition/subtraction and verify with technology.
Prerequisite Skills	• Students should know the exact values of sine, cosine and tangent from the angles in the unit circle.
Connections Within and Beyond High School	• The techniques used in proving these formulas can be helpful with verifying other trigonometry identities. • Proving double angle and half angle formulas.
Instructional Examples/Lessons/Tasks	• Trigonometry and Angles Notes Part I (RPDP) • Trigonometry and Angles Notes Part II (RPDP) • Algebra II, Module 2, Topic B, Lesson 17 (engage NY) • Sample Assignments and Aligned Resources (shmoop)
Assessment Examples	• Trigonometry and Angles Part I Practice Test (RPDP) • Trigonometry and Angles Part II Practice Test (RPDP)

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