Mathematics Standards Clarification for Statistics & Probability Conceptual Category

High School

2019

The Nevada Ready! Network

Designed for teachers by teachers!

The Nevada Ready! Network

Standards-Based Instruction for ALL Nevada Students

Contents

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on a single count or measurement variable

NVACS HSS.ID.A.1 (Major Supporting Work)

Represent data with plots on the real number line (dot plots, histograms, and box plots).

Element	Exemplars
Standards for Mathematical Practice	● MP 4: Students will be able to identify important quantities in practical situations and map their relationships using graphs. ● MP 5: Students will use appropriate mathematical tools, like pencil and paper, computer software, and graphing calculators to present data in graphs.
Instructional Strategies	● Begin by showing students how to place numerical data in a dot plot with an appropriate number line and title. ○ A fun way to do dot plots is give each student a sticky note and have them write the number of letters in their first name on the paper. They then come up to the board and create a dot plot with their sticky notes. ● Students can make a boxplot or histogram using their heights. This will get students out of their seats and reinforce concepts in the two graphical displays. RPDP-Math High School Go to Algebra I Resources, click on Unit 1 Descriptive Data and Activities and then open Data Analysis Activities. ● Show students how to group data into classes to create a frequency table. Make sure to discuss differences in classes when dealing with discrete and continuous data. ○ Computer software, like Excel or Shodor, can be used to create frequency tables and bar graphs. ○ Focus on technology when using larger data sets. ○ Applets can also be used to construct graphs quickly. Applets- Stats ● Make sure to explain the differences between bar graphs (discrete data) and histograms (continuous data). ● Introduce the 5-number summary to students and create box plots with and without showing outliers. Explain that the 5-number summary divides the data into equal quarters.
Prerequisite Skills	● Classify data as either categorical or quantitative. ● Understand the difference between discrete and continuous quantitative variables. ● Identify an appropriate scale needed for the data display.
Connections Within and Beyond High School	● To be an informed citizen and be able to dissect information in a technological world.

Element	Exemplars
Instructional Examples/Lessons/Tasks	• Histograms and Bar Graphs Introduction Lesson Histograms and Bar Graphs Intro Lesson (Shodor Interactivate Lessons) • Quartiles and Box Plots Quartiles and Box Plots (Shodor Interactivate Lessons) • Speed Trap Speed Trap (Illustrative Mathematics) • How to create data displays How to create data displays (Online Statbook)
Assessment Examples	• RPDP-Math High School Go to Algebra I Resources, click on Unit 1 Descriptive Data and then open the Practice Test

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on a single count or measurement variable

NVACS HSS.ID.A.2 (Major Supporting Work)

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Students make sense of quantities in graphs and their relationships using shape, center and spread. ● MP 6: Students communicate to others using appropriate vocabulary about the differences in data sets and how precise the differences are.
Instructional Strategies	● Explain the differences between mean (arithmetic average) and median (middle number) and which is more appropriate to report depending on the shape of the data. For symmetric data, we use the mean and for skewed data the median is more appropriate for use. ● Students need to be familiar with symmetric and skewed data and how the measures of center are affected by the shape of the data distribution. StatTrek - Stats Data Patterns (StatTrek) ● The measures of variability that students should be familiar with are interquartile range (IQR) and standard deviation. Make sure to show students how to use the standard deviation formula so they can understand that the standard deviation is the average amount that the data varies from the mean. Once the definition is understood, however, technology should be used to compute the standard deviation. ○ Scientific calculators, graphing calculators and applets can find the standard deviation. Applet stats MathisFun Standard Deviation Calculator NC Calculators mean standard deviation calculator ● When the data is symmetric, the standard deviation is reported along with the mean as the measure of central tendency. When data is skewed, the interquartile range is reported along with the median. ● When comparing data sets, students should use comparison words like smaller, larger, etc. We should compare the centers and spreads by describing which one is more extreme than the other.

Element	Exemplars
Prerequisite Skills	• Concepts of center: mean, median and mode from statistics standards seen in the 6th grade. • Difference between center and spread. Variability (spread) is the average amount the data varies from the mean and students should be familiar with simple spread measures, like range. • Creating and interpreting dot plots, bar graphs, histograms and box plots.
Connections Within and Beyond High School	• This knowledge will be useful in interpreting data in real-world scenarios.
Instructional Examples/Lessons/Tasks	• Three lessons on describing shapes and centers of data Describing Shapes and Centers of Data - Engage NY (Engage NY Algebra 1 Module 2 Topic A) • Five lessons on describing and variability and comparing distributions Describing and Variability and comparing distributions (Engage NY Algebra 1 Module 2 Topic B) • Haircut Costs Haircut Costs (Illustrative Mathematics) • Measuring Variability in a Data Set Measuring Variability in a Data set (Illustrative Mathematics) • Analyzing One-Variable Data Analyzing One-Variable Data (Curriculum Pathways Algebra I Course Unit 5 Data Analysis) • Measures of Central Tendency Measures of Central Tendency (MathBitsNotebook)
Assessment Examples	• RPDP- Math High School Go to Algebra I Resources, Unit 1 Descriptive Data, then Practice Test • Practice interpreting graphs Practice interpreting graphs (MathbitsNotebook)

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on a single count or measurement variable

NVACS HSS.ID.A.3 (Major Supporting Work)

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students analyze graphs by explaining the shape, center and spread in data sets and search for similarities and differences or trends. ● MP 2: Students make sense of quantities in graphs and their relationships using shape, center and spread. ● MP 6: Students communicate to others using appropriate vocabulary about the differences in data sets and how precise the differences are.
Instructional Strategies	● Explain how the mean and median are affected by extreme data points. ● To help the students understand how the mean and median are affected by outliers, this can be done best with online applets. ○ Mean and median- Applet ○ NCTM - Interactives Mean and Median ● Students need to be familiar with symmetric and skewed data and how the measures of center are affected by the shape of the data distribution. ○ Shape of the data distribution (StatTrek) ○ Variability simulation: Variability simulation (OnelineStatbook) ● For the measures of variability, the standard deviation is not resistant and is also greatly affected by outliers. Interquartile range is not affected by outliers. ○ Scientific calculators, graphing calculators and applets can find the standard deviation. Applet calculator MathisFun-Standard Deviation calculator NC Calculator - Standards Deviation Calculator
Prerequisite Skills	● Students should be familiar with the concepts of center: mean, median and mode from statistics standards seen in the 6th grade. ● Students should be familiar with the difference between center and spread. Variability (spread) is the average amount the data varies from the mean and students should be familiar with simple spread measures, like range. ● Students should be familiar with creating and interpreting dot plots, bar graphs, histograms and box plots.

Element	Exemplars
Connections Within and Beyond High School	• To be an informed citizen and be able to dissect information in a technological world.
Instructional Examples/Lessons/Tasks	• Three lessons on describing shapes and centers of data Describing Shapes and Centers of Data (Engage NY Algebra 1 Module 2 Topic A) • Five lessons on describing and variability and comparing distributions Describing and variability and comparing distributions (Engage NY Algebra 1 Module 2 Topic B) • Describing Data Sets with Outliers Describing Data Sets with Outliers (Illustrative Mathematics) • Identifying Outliers Identifying Outliers (Illustrative Mathematics) • Measuring Variability in a Data Set Measuring Variability in a Data Set (Illustrative Mathematics) • Graphing One-Variable Data Graphing one - Variable Data (Curriculum Pathways Algebra I Module)
Assessment Examples	• RPDP - Math High School Go to Algebra I Resources, Unit 1 Descriptive Data, then Practice Test

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on a single count or measurement variable

NVACS HSS.ID.A.4 (Major Supporting Work)

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students will assess the fit of the normal distribution to a set of data and use the model to find population percentages as well as working backwards to find values. ● MP 4: Students will model the normal distribution for data sets where the model is appropriate. Students will determine whether the normal distribution can be used to find population percentages. ● MP 5: Use appropriate tools like calculators, spreadsheets, tables and software to find the area under the normal distribution.
Instructional Strategies	● Normal distributions are unimodal, symmetric, bell-shaped distributions that vary in shape depending on the standard deviation. Students should be able to find a z-score and interpret it. Notes to help introduce the normal distribution can be found at MVP - Algebra II, Module 9, lessons 9.1–9.4 Mathematics Vision Project - Algebra II ● Looking at histograms and determining if they are normal is a difficult skill and can be modeled with different online resources: ○ Shodor - Histograms ● Video to introduce the topic: Normal Distribution Curve Video ● Introduce the Empirical rule with the following notes which also has a normal distribution calculator: Probability Distributions calculator ● A fun activity to use with the Empirical Rule is the Normal Distribution and Empirical Rule Pop-Up: Empirical Rule Pop-up ● There are many tools online to help calculate percentages under the normal curve as well as graphing calculators. ○ normal online stat calculator ○ Applet calculator (Nice visual) ○ David Lane Hyperstat visual ○ Mathbits normal distribution calculator ● Students should also be able to calculate the inverse normal distribution when given a percentage. Inverse norm calculators can also be found online as well as on graphing calculators. ○ Normal distribution calculator ○ Calculator ○ Mathbits Normal Distribution Calculator
Prerequisite Skills	● Mean and standard deviation. ● Distribution shapes, center, and spread.
Connections Within and Beyond High School	● Many data distributions in the real world approximate normal distributions. Knowledge of this topic will help students to understand their data-filled world.
Instructional	● Lesson on when a normal distribution is appropriate

Element	Exemplars
Examples/Lessons/Tasks	Normal Distribution lesson (Engage NY, Algebra II, Module 4, Topic B, Lesson 9) ● Calculating z-scores and interpreting probabilities Calculating Z-scores and interpreting probabilities (Engage NY, Algebra II, Module 4, Topic B, Lesson 10) ● Final lesson normal probabilities Final Lesson normal probabilities (Engage NY, Algebra II, Module 4, Topic B, Lesson 11) ● Do You Fit in this Car? Do You Fit in this Car? (Illustrative Mathematics) ● SAT Scores SAT Scores lesson (Illustrative Mathematics) ● Should We Send Out a Certificate? Should We Send Out a Certificate lesson (Illustrative Mathematics)
Assessment Examples	● Practice with Normal Distributions (MathBits Algebra 2, Statistics, Practice with Normal Distributions) ● eMathInstruction practice (eMathinstruction, Algebra 2, Unit 13 - Lesson 3 and 4 pdfs have assessment questions) ● Practice with z-scores (MathBits Algebra 2, Statistics, Practice with z-scores) ● Applications of z-scores ● (MathBits Algebra 2, Statistics, Applications of z-scores)

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on two categorical and quantitative variables

NVACS HSS.ID.B.5 (Major Supporting Work)

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students will be able to analyze two-way tables and make conjectures about possible associations regarding the two categories. ● MP 4: Students will be able to summarize categorical data for two categories using the tool of two-way tables to map their relationship. They can analyze the relationship and draw conclusions about possible trends in the data.
Instructional Strategies	● The vocabulary needs to be established at the beginning. Make sure to explain the differences between frequencies (counts) and relative frequencies (fraction or decimal of the total) as well as joint frequencies, marginal frequencies and conditional frequencies. ○ The following website defines the above terms: Stattrek tutorial ● Find two-way tables with categories that are interesting to the students and have them interpret frequencies, but also have them create their own two-way tables. The following activity can be used: ○ Two-way tables activity ● A data set that can be fun to use is the Titanic data. This data in two-way tables can be found on many websites and many include activities for the classroom: ○ Titanic two-way frequency tables ○ Two-way tables activity ○ Two-way table of counts and graphs ● Students should also come to understand the distinction between association between two categorical variables and a causal relationship between two variables. This provides a foundation for work on sampling and inference in later grades.
Prerequisite Skills	● Difference between categorical and quantitative data. ● Difference between univariate and bivariate data. ● Conversions between fractions, decimals, and percents.
Connections Within and Beyond High School	● Distinguishing between correlation and causation ● Determining the dependence or independence of two variables

Element	Exemplars
Instructional Examples/Lessons/Tasks	• Three lessons on Categorical Data on Two Variables Engage NY Categorical data on two variables (Engage NY Algebra I, Module 2, Topic C) • Two-Variable Categorical Data Two-variable categorical data (Curriculum Pathways Algebra I Course Unit 5) • Musical Preferences Musical Preferences (Illustrative Mathematics) • Support For a Longer School Day? Support for a Longer School Day (Illustrative Mathematics) • Two-Way Tables Census Bureau Two-way tables census bureau
Assessment Examples	• Two-Way Frequency Tables Two-way Frequencey Tables (MathBitsNotebook) • Practice Questions eMathinstruction practice questions (eMathinstruction) Go to Lesson 5 and open the PDF lesson

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on two categorical and quantitative variables

NVACS HSS.ID.B.6 (Major Supporting Work)

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students will be able to discuss important features of the scatter plot and explore the relationship between the two variables using the graph. ● MP 4: Students model bivariate data relationships by creating scatter plots. ● MP 5: Students will use paper, pencil and technology, when appropriate to create their scatterplots.
Instructional Strategies	● Vocabulary is very important and students should know the following terms: dependent and independent variable, positive, negative and no correlation, weak, moderate and strong correlation, and outliers. ● Use the following Desmos activity to help students see the difference between positive, negative and no correlation as well as weak, moderate or strong. Desmos polygraph activity ● Create scatter plots by hand, but also use technology like graphing calculators, Desmos, Excel or other applets and programs. ○ Two-Variable Statistical Calculator: Two-variable statistical calculator ○ Scatter plot on Shodor: Scatter Plot Shodor ○ Meta Chart: Meta Chart Scatter
Prerequisite Skills	● Bivariate data ● Difference between categorical and quantitative bivariate data. ● Difference between independent and dependent variables. ● Creating graphs, labeling axes, and picking appropriate scaling.
Connections Within and Beyond High School	● Using scatterplots to make predictions by finding the line of best fit. ● Finding associations between bivariate quantitative data.

Element	Exemplars
Instructional Examples/Lessons/Tasks	• Great activity to introduce scatterplots Introduce Scatterplots (Mathbits) • Two-Variable Quantitative Data Two-variable quantitative data (Curriculum Pathways Algebra I Course, Unit 5) • Scatterplots lesson Scatterplots lesson (Stattrek) • Bivariate Data Analysis Bivariate Data Analysis (eMathinstuction) Go to the pdf lesson in Lesson 6 • Star Scatter Plots Star Scatter Plots (CPalms) • Used Suburu Foresters I Used Subaru Foresters activity (Illustrative Mathematics) • Scatter plot Capture Scatter Plot Capture (Desmos Activities)
Assessment Examples	• What's Your Association? What's your association? activity (CPalms)

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on two categorical and quantitative variables

NVACS HSS.ID.B.6A (Major Supporting Work)

Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Element	Exemplars
Standards for Mathematical Practice	• MP 1: Students make sense of problems choosing functions fitted to data. • MP 2: Students will reason abstractly and quantitatively to determine which function will fit the data. • MP 6: Students will communicate precisely to others about why the function represents the data. • MP 7: Students will determine which function is the best fit for the data
Instructional Strategies	• Graphing Calculator/GeoGebra/Excel • Hands-on data collection
Prerequisite Skills	• Create a function table. • Create a scatterplot. • Observe trends in data.
Connections Within and Beyond High School	• Use technology to analyze patterns and describe relationships between two variables in context. • Analyze patterns and describe relationships between variables in context.
Instructional Examples/Lessons/Tasks	• Used Suburu Foresters I Used Subaru Foresters activity (Illustrative Mathematics) • Coffee & Crime Coffee and Crime (Illustrative Mathematics) • Olympic Men's 100m Dash Olympic Mens' 100m Dash (Illustrative Math) • Laptop Battery Charger 2 Laptop Battery Charger 2 activity (Illustrative Math)
Assessment Examples	• Regression PreReview (Quizizz)

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on two categorical and quantitative variables

NVACS HSS.ID.B.6B (Major Supporting Work)

Informally assess the fit of a function by plotting and analyzing residuals.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Students make sense of the linear relationship by analyzing the residuals. ● MP 3: Students will reason whether the function is appropriate by analyzing the residuals.
Instructional Strategies	● Introduce students to residuals by defining it as the vertical distance between the actual or observed *y* value in the data to the predicted *y* value calculated by the line of best fit. These residuals can help us to determine whether the line of best fit is appropriate by constructing a residual plot and noting its pattern: Stattrek dictionary ● The residuals can be shown in the Analyzing Two Quantitative Variables Applet and on Shodor: ○ RegShuffle Click on "Show Regression Line" and then "Show Residuals" ○ Regression - Shodor ● Residual plots can be drawn by hand, but after doing one in this manner, technology should be used as it can be tedious. Graphing calculators, Desmos and applets can create residual plots. ○ Desmos calculator ○ Two Variable Statistical Calculator: ○ For instructions on creating residual plots on the TI-84: ● Show students several different residual plots and ask them if the model is a good fit. Students should be looking for random scatter in the plot and avoid any curvature or pattern. Stattrek Regression analysis
Prerequisite Skills	● Be familiar with bivariate data and constructing and interpreting scatter plots. ● Be familiar with finding the line of best fit.
Connections Within and Beyond High School	● Reinforcement of skills in assessing evidence and drawing a conclusion.
Instructional Examples/Lessons/Tasks	● Restaurant Bill and Party Size (Illustrative Mathematics) ● Residual Plots lesson (Engage NY, Algebra I, Module 2) ● Residual Plots lesson (Engage NY, Algebra I, Module 2)
Assessment Examples	● Practice with residuals ● Quizizz (free account)

Interpreting Categorical and Quantitative Data

Cluster

Summarize, represent, and interpret data on two categorical and quantitative variables

NVACS HSS.ID.B.6C (Major Supporting Work)

Fit a linear function for a scatter plot that suggests a linear association.

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students must recognize when a linear relationship is present in a scatter plot. ● MP 4: Students will analyze the given scatter plot and using the process of least squares regression, create a line of best fit. This model will then represent the relationship shown in the scatter plot. ● MP 7: Students will have to discern the pattern and structure in the scatter plot to create the linear model.
Instructional Strategies	● Define what is meant by "Line of best fit" ● Students should be able to create the regression line by hand, using their "eye", by using a ruler and fitting their perfect line, finding two points on their line and creating the linear equation. However, students should also be able to find the regression line using technology. Graphing calculators, Desmos and applets can find the least squares regression line. ○ NC Calculator Linear regression calculator ○ Desmos calculator ○ Line fit calculator ○ Applets for squares regression line ● After finding the line of best fit, students should be able to use their model to make predictions within the domain of their data. Avoid extrapolation by making predictions outside our observed values.
Prerequisite Skills	● Be familiar with scatterplots and types of association. ● Be familiar with linear functions and their vocabulary: slope, rate of change, *y*-intercept. ● Graph a line and find the equation of a line given two points.
Connections Within and Beyond High School	● Students will be using their knowledge of linear functions in order to select best fit data functions.

Element	Exemplars
Instructional Examples/Lessons/Tasks	• Least Squares Regression Lesson (Engage NY Algebra I, Module 2) • Line of Best Fit Activity (Desmos Activity) • Linear Regression and Correlation (Shodor) • Linear Regression on the Calculator (eMATHinstruction, Lesson 7) • Two-Variable Quantitative Data (Curriculum Pathways, Algebra I Course, Unit 5.4) • Linear Regression tasks (MVP, Module 9)
Assessment Examples	• RPDP practice test Algebra I Resources, Unit 6, Practice Test • Practice with Linear Regression

Interpreting Categorical and Quantitative Data

Cluster

Interpret linear models

NVACS HSS.ID.C.7 (Major Supporting Work)

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Element	Exemplars
Standards for Mathematical Practice	• MP 6: Students are communicating precisely with clear definitions, regarding the parameters of a linear function.
Instructional Strategies	• Stress the importance of the student definitions to be in the context of the data. • Graph two different situations to compare/contrast the slope and *y*-intercept within the different contexts.
Prerequisite Skills	• Find the rate of change graphically and algebraically. • Find the *y*-intercept graphically and algebraically. • Find the equation of a line using data points.
Connections Within and Beyond High School	• Interpreting parameters in functions, including but not limited to linear, quadratic, exponential and rational.
Instructional Examples/Lessons/Tasks	• Slopes and Intercepts of linear models (MathBits) • Texting and Grades II (Illustrative Mathematics) • Olympic Men's 100-meter Dash (Illustrative Mathematics) • Slope and *y*-intercept of a Statistical Model (Shodor)
Assessment Examples	• Practice with Slope and Intercepts in Linear Regression • Used Suburu Foresters II (Illustrative Mathematics)

Interpreting Categorical and Quantitative Data

Cluster

Interpret linear models

NVACS HSS.ID.C.8 (Major Supporting Work)

Compute (using technology) and interpret the correlation coefficient of a linear fit.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will make sense of the correlation coefficient and be able to interpret the number in terms of the relationship between the two variables. • MP 6: Students will compute the correlation coefficient and communicate precisely to others about what this measure explains about our data.
Instructional Strategies	• The students will only use technology to find the correlation coefficient. The correlation coefficient can be found on graphing calculators, Desmos and other applets. Below is a list of places for students to find r, the correlation coefficient. ○ Applets Reg Shuffle ○ Desmos calculator ○ Finding r on the TI-84 graphing calculator: • To make the topic interesting, have the students play the Guess the Correlation Game. They are shown scatterplots and have to guess the correlation coefficient. • When interpreting the correlation coefficient, use words like positive, negative and strong, moderate or weak. There are no stringent rules on the boundaries for weak, moderate or strong, but the following can be used as a guideline: TheCorrelationCoefficientDefined • Use the following website to help define and explain the correlation coefficient: Correlation Coefficients website
Prerequisite Skills	• Linear relationship and how to find the least squares regression line. • Interpret scatter plots.
Connections Within and Beyond High School	• Students will use the knowledge of the correlation coefficient to understand the strength or weakness of their predictions based on the linear regression equation.
Instructional Examples/Lessons/Tasks	• Guess the Correlation Applet (OnlineStatbook) • Plot points and find residuals and the correlation coefficient (Statistics applet) • Correlation Guessing Game (Rossman/Chance applets) • Regression by Eye applet (Java is needed to run this applet)
Assessment Examples	• Practice problems with correlation coefficient on Mathbits

Interpreting Categorical and Quantitative Data

Cluster

Interpret linear models

NVACS HSS.ID.C.9 (Major Supporting Work)

Distinguish between correlation and causation.

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Students will justify their conclusions, communicate them to others, and respond to the arguments of others by distinguishing between correlation and causation of variables. • MP 7: Students will assess the pattern of two quantitative variables and make decisions based on the structure if there is correlation or causation present.
Instructional Strategies	• Great way to introduce the topic is to use the following article: Correlation vs. Causation - Everyday Einstein: Quick and Dirty Tips for Making Sense of Science • Another article that is helpful for introducing the two terms is in Towards Data Science • Bring in science applications of correlation and causation. • Videos to show students the differences between causation and correlation
Prerequisite Skills	• Linear relationships • Interpret the correlation coefficient.
Connections Within and Beyond High School	• To discern relationships between variables in a data filled world.
Instructional Examples/Lessons/Tasks	• Curriculum Pathways helps students distinguish between the two terms in its lesson - Algebra I, Unit 5-4 (Curriculum Pathways) • Towards Data Science article great for the classroom. (Towards data Science) • Spurious Correlations - funny examples! (Tylervigen)
Assessment Examples	• Correlation and Causation.pdf

Making Inferences and Justifying Conclusions

Cluster

Understand and evaluate random processes underlying statistical experiments

NVACS HSS.IC.A.1 (Major Supporting Work)

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Students will reason inductively about data, making plausible arguments that take into account the context from which the data arose and make inferences about parameters based on statistics. • MP 4: Students will make assumptions and approximations about parameters based on sample data.
Instructional Strategies	• Vocabulary: Clearly define parameters (population characteristics) and statistics (sample characteristics). Discuss the ways for collecting data, including random and nonrandom procedures to illustrate the importance of randomness. The following activity is useful for explaining the importance of randomizing: ○ Random Sampling activity ○ Random Rectangles Activity ○ StatsMonkey Activities • To better understand the intent of the standard • The students should be familiar with the following types of sampling: Voluntary Response survey, Convenience sampling, Simple Random Sample (SRS), Systematic random sample, Stratified random sample and Cluster sample. These are defined in the following website: Sampling Methods - Stattrek
Prerequisite Skills	• Statistical measures, like proportions and means. • Distributions, shape, center and spread.
Connections Within and Beyond High School	• Making inferences and justifying conclusions about data from surveys, experiments and studies.
Instructional Examples/Lessons/Tasks	• Musical Preferences (Illustrative Mathematics) • Strict Parents • Why Randomize? (Illustrative Mathematics) • Engage NY, Algebra 2, Module 4, Topic C, Lesson 13 • Engage NY, Algebra 2, Module 4, Topic C, Lesson 12
Assessment Examples	• School Advisory Panel (Illustrative Mathematics)

Making Inferences and Justifying Conclusions

Cluster

Understand and evaluate random processes underlying statistical experiments

NVACS HSS.IC.A.2 (Major Supporting Work)

Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*?

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students will determine whether empirical results are consistent with the theoretical model. ● MP 4 Students will be using simulations to generate data to explore randomness.
Instructional Strategies	● Students should be able to generate random numbers using a random number table, calculator or other applet or computer. ○ Random Samples And Random Permutations ○ Random Number generator ○ Simple random sample sampler: ● There should be hands-on activities with students doing simulations to illustrate data-generating processes like, SRS, stratified and systematic. Use the website random.org to generate flipping coins, rolling dice, picking, cards, etc. if those tools are not available. ● This applet will generate a sample of Reese's Pieces to illustrate a random sample from a population.
Prerequisite Skills	● Create a probability distribution table and interpret tables. ● Histograms and their properties.
Connections Within and Beyond High School	● Making inferences and justifying conclusions about data from surveys, experiments and studies.
Instructional Examples/Lessons/Tasks	● Block Scheduling (Illustrative Mathematics) ● Guess the Probability (Illustrative Mathematics) ● Last Person Standing (Illustrative Mathematics) ● Sarah, the Chimpanzee (Illustrative Mathematics)
Assessment Examples	● Khan Academy Questions

Making Inferences and Justifying Conclusions

Cluster

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

NVACS HSS.IC.B.3 (Major Supporting Work)

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Students will reason abstractly by recognizing the purposes and differences among the ways data can be collected. ● MP 4: Students will model the different types of data-gathering techniques and illustrate how they can be used in real-world applications.
Instructional Strategies	● Students should be given different situations and asked to decide whether the data should be gathered using an observational study, through experiment or through a sample survey. They should be familiar with the pros and cons of each type. ○ STSurveys ○ STSurvey Practice ○ Stattrek data-collection-methods ● More notes and questions to use with the students to identify the type of data-gathering technique used. Common Core Standards ● Bias in survey sampling should be discussed - the types and the ways to avoid them. ○ Survey Bias ○ Research Design Sampling ○ Real world examples of bias in surveys ● Two great activities illustrating the different types of sampling and their biases: Choose the two activities near the bottom of the page - It's Elemental! Sampling from the Periodic Table, or Sampling in Archaeology
Prerequisite Skills	● Randomization and bias ● Parameters and statistics. ● Display data distributions.
Connections Within and Beyond High School	● To become better statistically aware citizens in a data-driven world. ● Drawing conclusions from sample data to make inferences about population parameters.

Element	Exemplars
Instructional Examples/Lessons/Tasks	• High Blood Pressure (Illustrative Mathematics) • Strict Parents (Illustrative Mathematics) • Words & Music II (Illustrative Mathematics) • Lesson on data-gathering techniques (Engage NY, Algebra 2, Module 4, Topic C, Lesson 12) • Double Down (TI Calculator Activity) • Randomization in Sampling and Experiments (TI Calculator Activity) • Video Lesson on Selection Bias MIT Blossoms
Assessment Examples	• Practice Multiple-choice questions (Khan Academy Questions on S.IC.B.3) • Types of Statistical Studies Questions (Illustrative Mathematics)

Making Inferences and Justifying Conclusions

Cluster

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

NVACS HSS.IC.B.4 (Major Supporting Work)

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Element	Exemplars
Standards for Mathematical Practice	● MP 4: Students will model sampling distributions of means and proportions through simulation to estimate the parameters. ● MP 5: Students will use appropriate tools like paper and pencil, tables, software and graphing calculators to develop a margin of error to estimate population means and proportions.
Instructional Strategies	● Students need to have a basic understanding of a margin of error. ● Estimations of means and proportions: ○ Using Dice to Introduce Sampling Distributions Activity Who Sends the Most Text Messages? Population Parameter with M&M's ● Margin of error: ○ Simulating Confidence Intervals applet: ○ One proportion inference applet: ● Confidence intervals for proportions and the confidence intervals: ● Directions for finding the margin of error along with practice problems ● Lessons can be found in Engage NY on margin of error and estimating population means and proportions. Look at Lessons 16–21 in Topic C, Module 4 in Algebra II. ● Lessons on Sample means and Sample proportions can be found at Open the pdfs for Lesson 5 and Lesson 6.
Prerequisite Skills	● Understanding of basic probabilities. ● Parameters and statistics and how to find averages and proportions of data distributions. ● How to construct a dot plot/histogram/boxplot and how to informally describe shape, center and spread. ● Calculator the mean and standard deviation of a distribution. ● Use the normal distribution table to find critical values.
Connections Within and Beyond High School	● Evaluate reports, conclusions and hypotheses based on data.

Element	Exemplars
Instructional Examples/Lessons/Tasks	• Fred's Flare Formula (Illustrative Mathematics) • The Marble Jar (Illustrative Mathematics) • Margin of Error and Sample Size (TI Calculator Activity) • Estimating a Population Proportion (TI Calculator Activity) • Margin of Error and Sample Size (TI Inspire Activity) • Mathbits Algebra II
Assessment Examples	• Margin of Error for Estimating a Population Mean (Illustrative Mathematics) • Scratch & Win Blues (Illustrative Mathematics) • Practice Problems on Margin of Error (jmap.org)

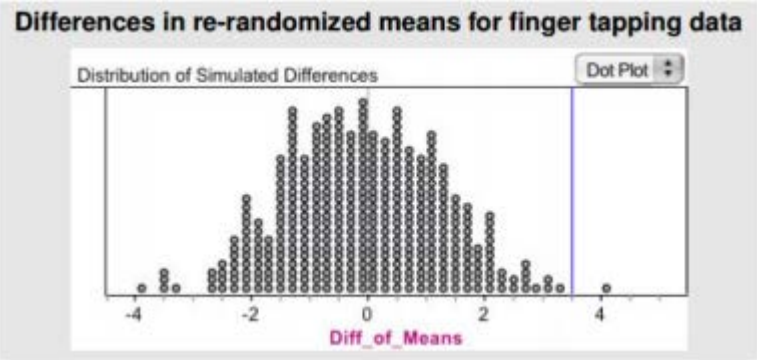
Making Inferences and Justifying Conclusions

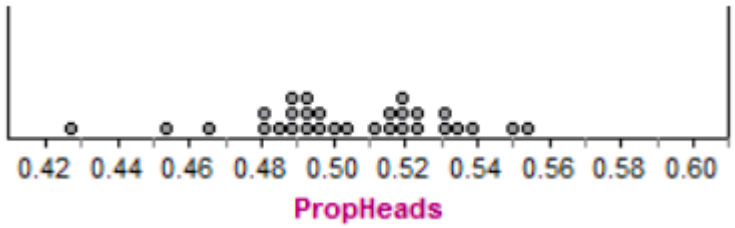
Cluster

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

NVACS HSS.IC.B.5 (Major Supporting Work)

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Element	Exemplars
Standards for Mathematical Practice	• MP 4: Students use real simulation data to examine the significance of parameters.
Instructional Strategies	• Although not identified as a standard for an advanced course, this standard is more appropriate for an advanced course.
Prerequisite Skills	• Design and use simulation to generate frequencies for compound events. • Understand the process of making inferences.
Connections Within and Beyond High School	• Recognize the purpose and differences between samples and studies and how randomization is used. • Use simulation to estimate a population mean or proportion.
Instructional Examples/Lessons/Tasks	• Consider the experiment where twenty male students were randomly assigned to one of two treatment groups of 10 students each, one group receiving 200 milligrams of caffeine and the other group no caffeine. The parameter of interest is the number of finger taps per minute. The sample statistics showed that the mean of the 200 mg group was 3.5 taps more than the 0 mg group. Thus, an observed difference. Use simulation to determine if the observed difference is due to the caffeine. Is it possible that the 3.5 taps was due to randomization and not caffeine? In order to find out, re-randomize the participants and calculate the difference in means. Simulate this and create a distribution of the results.  (The results of the simulation shows that the difference of 3.5 is equaled or exceeded only once out of 400 trials this providing strong evidence that the caffeine is the cause of the increased tapping.)

Element	Exemplars
Assessment Examples	• Sal purchased two types of plant fertilizer and conducted an experiment to see which fertilizer would be best to use in his greenhouse. He planted 20 seedlings and used Fertilizer A on ten of them and Fertilizer B on the other ten. He measured the height of each plant after two weeks. Use the data below to determine which fertilizer Sal should use. Fertlizer A 23.4 30.1 28.5 26.3 32.0 29.6 26.8 25.2 27.5 30.8 Fertlizer B 19.8 25.7 29.0 23.2 27.8 31.1 26.5 24.7 21.3 25.6 a. Use the data to generate simulated treatment results by randomly selecting ten plant heights from the twenty plant heights listed. b. Calculate the average plant height for each treatment of ten plants. c. Find the difference between consecutive pairs of treatment averages and compare. Does your simulated data provide evidence that the average plant heights using Fertilizer A and Fertilizer B is significant? • "Are Starbucks customers more likely to be female?" To answer the question, students decide to randomly select 30-minute increments of time throughout the week and have an observer record the gender of every tenth customer who enters the Starbucks store. At the end of the week, they had collected data on 260 customers, 154 females and 106 males. This data seems to suggest more females visited Starbucks during this time than males. To determine if these results are statistically significant, students investigated if they could get this proportion of females just by chance if the population of customers is truly 50% females and 50% males. Students simulated samples of 260 customers that are 50-50 females to males by flipping a coin 260 then recording the proportion of heads to represent the number of women in a random sample of 260 customers (e.g., 0.50 means that 130 of the 260 flips were heads). Their results are displayed in the graph.  Use the distribution to determine if the class's data is statistically significant enough to conclude that Starbucks customers are more likely to be female.

Making Inferences and Justifying Conclusions

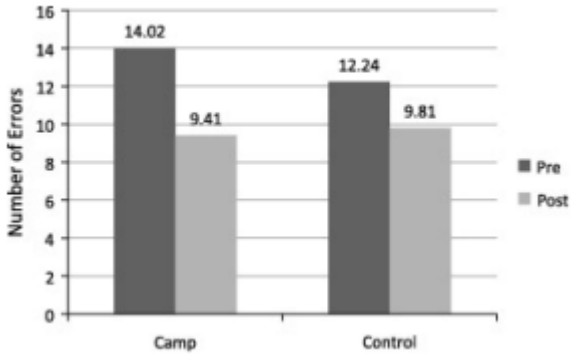
Cluster

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

NVACS HSS.IC.B.6 (Major Supporting Work)

Evaluate reports based on data.

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Students will interpret reports and analyze the validity of statements resulting from those reports. Students will construct arguments in favor or opposing statements based on data using appropriate assumptions and definitions. • MP 5: Students will select data displays based on the given situation.
Instructional Strategies	• Give the students a set of data and have them decide what type of display would be appropriate, and have them make that display. • Have students with different types of displays try to convince the class why there type is superior. • Students should recognize how graphs and data can be distorted to support different points of view.
Prerequisite Skills	• Data displays such as circle graphs, bar graphs, histograms, line plots, box-whisker plots, etc. • Center and spread of two or more data sets and interpret differences in context. • Recognize the purpose and differences between samples and studies and how randomization is used.
Connections Within and Beyond High School	• Students should use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays. • Evaluating appropriateness of different data displays is a central part of data applications in many real world fields
Instructional Examples/Lessons/Tasks	• EngageNY CCLS Math
Assessment Examples	• Read the article below from NPR.org then answer the following questions. *Kids and Screen Time: What Does the Research Say? By Juana Summers August 28, 2014.* Kids are spending more time than ever in front of screens, and it may be inhibiting their ability to recognize emotions, according to new research out of the University of California, Los Angeles. The study, published in the journal Computers in Human Behavior, found that sixth-graders who went five days without exposure to technology were significantly better at reading human emotions than kids who had regular access to phones, televisions and computers. The UCLA researchers studied two groups of sixth-graders from a Southern California public school. One group was sent to the Pali Institute, an outdoor education camp in Running Springs, Calif., where the kids had no access to electronic devices. For the other group, it was life as usual. At the beginning and end of the five-day study period, both

Element	Exemplars
	groups of kids were shown images of nearly 50 faces and asked to identify the feelings being modeled. Researchers found that the students who went to camp scored significantly higher when it came to reading facial emotions or other nonverbal cues than the students who continued to have access to their media devices. "We were pleased to get an effect after five days," says Patricia Greenfield, a senior author of the study and a distinguished professor of psychology at UCLA. "We found that the kids who had been to camp without any screens but with lots of those opportunities and necessities for interacting with other people in person improved significantly more." If the study were to be expanded, Greenfield says, she'd like to test the students at camp a third time–when they've been back at home with smartphones and tablets in their hands for five days. "It might mean they would lose those skills if they weren't maintaining continual face-to-face interaction," she says.  a. What is the source of the data? b. Describe the design of the study. c. After analyzing the graph, evaluate the claim that the “kids who had been to camp improved significantly more.”

Conditional Probability and the Rules of Probability

Conditional Probability & the Rules of Probability

Cluster

Understand independence and conditional probability and use them to interpret data

NVACS HSS.CP.A.1 (Major Supporting Work)

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Element	Exemplars
Standards for Mathematical Practice	• MP 5: Students will be able to choose an appropriate organizational tool such as a Venn diagram to organize elements of a sample space. • MP 7: Students will be able to model the structure of a set of elements and does or does not share elements with other sets.
Instructional Strategies	• Use Venn diagrams to model sets, subsets, intersections, unions & complements.
Prerequisite Skills	• Find probabilities of compound events using lists, tables, tree diagrams and simulations.
Connections Within and Beyond High School	• Set notation. • Use the rules of probability to compute probabilities & conditional probabilities. • These concepts are revisited and used in Statistics and higher level Algebras.
Instructional Examples/Lessons/Tasks	• Students define a sample space and events within the sample space. **Example:** Describe the sample space for rolling two number cubes. **Example:** Describe the sample space for picking a colored marble from a bag with red and black marbles. **Example:** A card is drawn from a deck of cards with the four aces removed, then a coin is tossed. Describe the sample space. • Students establish events as subsets of a sample space. An event is a subset of a sample space. **Example:** Describe the event of rolling two number cubes and getting evens. **Example:** Describe the event of pulling two marbles from a bag of red/black marbles. **Example:** Describe the event that the sum of two rolled number cubes is larger than 7 *and* even, and contrast it with the event that the sum is larger than 7 *or* even.
Assessment Examples	• Khan Academy HS CP

Conditional Probability and the Rules of Probability

Cluster

Understand independence and conditional probability and use them to interpret data

NVACS HSS.CP.A.2 (Major Supporting Work)

Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Students will conceptualize the difference between independent and dependent events. ● MP 3: Students will be able to defend the argument for independence or dependence of to events.
Instructional Strategies	● Use hands on examples and simulations with coins, dice, marbles, and cards including examples with or without replacement.
Prerequisite Skills	● Calculate simple probability.
Connections Within and Beyond High School	● This concept is essential to understanding the test for independence. ● This concept leads to the topic of causation/correlation in a statistics course.
Instructional Examples/Lessons/Tasks	● Math Goodies ● Khan Academy
Assessment Examples	● Students at the local high school were asked about their food preferences. Of the 437 students polled, 306 like hamburgers, 153 like chicken, and 47 like both. One student will be selected at random from the 437 students polled. ○ **Part A:** What is the probability that the person selected will like *neither* hamburgers nor chicken? ○ **Part B:** Let *H* represent the event that the student selected is one who likes hamburgers, and let *P(H)* represent the probability that event *H* will occur. Let *C* represent the event that the student selected is one who likes chicken, and let *P(C)* represent the probability that event *C* will occur. Which statement is true? A. Events *H* and *C* are independent because $P(H \text{ and } C) = P(H) \cdot P(C)$ B. Events *H* and *C* are independent because $P(H \text{ and } C) \neq P(H) \cdot P(C)$ C. Events *H* and *C* are **not** independent because $P(H \text{ and } C) = P(H) \cdot P(C)$ D. Events *H* and *C* are **not** independent because $P(H \text{ and } C) \neq P(H) \cdot P(C)$

Conditional Probability and the Rules of Probability

Cluster

Understand independence and conditional probability and use them to interpret data

NVACS HSS.CP.A.3 (Major Supporting Work)

Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

<table>
<tr><th>Element</th><th>Exemplars</th></tr>
<tr><td>Standards for Mathematical Practice</td><td>• MP 7: The test for independence is a specific formula structure that students need to be able to use.</td></tr>
<tr><td>Instructional Strategies</td><td>• Is often helpful to teach two-way tables before teaching this standard.</td></tr>
<tr><td>Prerequisite Skills</td><td>• Patterns of association from two-way tables in bivariate categorical data.
• Calculate simple probability.</td></tr>
<tr><td>Connections Within and Beyond Grade Level</td><td>• Probability formulas for “and” and “or” events.
• This concept leads to the topic of causation/correlation in a statistics course.</td></tr>
<tr><td>Instructional Examples/Lessons/Tasks</td><td>• Students can use two-way tables to find conditional probabilities.
Example: Each student in the Junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data.
<table><tr><td></td><td></td><td>Curfew</td><td></td><td></td></tr><tr><td></td><td></td><td>Yes</td><td>No</td><td>Total</td></tr><tr><td rowspan="3">Chores</td><td>Yes</td><td>51</td><td>24</td><td>75</td></tr><tr><td>No</td><td>30</td><td>12</td><td>42</td></tr><tr><td>Total</td><td>81</td><td>36</td><td>117</td></tr></table>
a. What is the probability that a student who has chores also has a curfew? b. What is the probability that a student who has a curfew also has chores? c. Are the two events have chores and have a curfew independent? Explain.</td></tr>
<tr><td>Assessment Examples</td><td>• A bag contains 3 red marbles and 4 blue marbles.
Two marbles are drawn at random without replacement.
If the first marble drawn is red, what is the probability the second marble is blue?
• Two cards are selected without replacement from a standard deck of 52. What is the probability that both are Aces?</td></tr>
</table>

Conditional Probability and the Rules of Probability

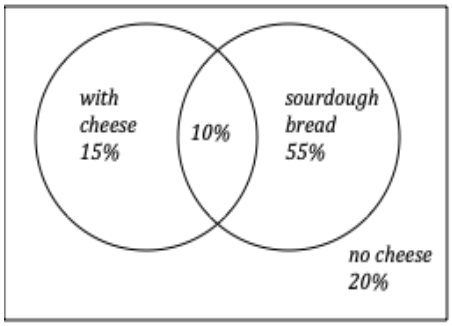
Cluster

Understand independence and conditional probability and use them to interpret data

NVACS HSS.CP.A.4 (Major Supporting Work)

Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4: Students should use real world data and examples to model how two-way tables function • MP 7: Students must be able to organize data using a two-way table demonstrating how elements can be represented using two characteristics.
Instructional Strategies	• Spend time building and interpreting two-way tables making connection to sets, subsets, and intersections. • Connect two-way tables to Venn diagrams to connect new learning to prior learning
Prerequisite Skills	• Understand patterns of association from two-way tables in bivariate categorical data.
Connections Within and Beyond High School	• Two-way tables can be used to present data to students as they learn how to use other probability formulas • These concepts are extended in advanced Statistics courses and apply to the real world.

Element	Exemplars
Instructional Examples/Lessons/Tasks	● Students can create a two-way frequency table for data and calculate probabilities from the table. **Example:** Collect data from a random sample of students in your school on their favorite subject among math, science, history, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. ● Students can use a two-way table to evaluate independence of two variables. **Example:** The Venn diagram to the right shows the data collected at a sandwich shop for the last six months with respect to the type of bread people ordered (sourdough or wheat) and whether or not they got cheese on their sandwich. Use the diagram to construct a two-way frequency table and then answer the following questions.  a. P (sourdough) b. P (cheese | wheat) c. P (without cheese or sourdough) d. Are the events “sourdough” and “with cheese” independent events? Justify your reasoning.
Assessment Examples	● Illustrative Math

Conditional Probability and the Rules of Probability

Cluster

Understand independence and conditional probability and use them to interpret data

NVACS HSS.CP.A.5 (Major Supporting Work)

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Students will use mathematics to prove/disprove premises concerning the independence of various events • MP 4: Students will apply conditional probability to everyday situations.
Instructional Strategies	• Try to find a current topic that students will know about and use data from this or have the students find the data for the class to use
Prerequisite Skills	• Basic probability rules • Tests for independence
Connections Within and Beyond High School	• This connects to S.MD.A and using probability to make decisions • Correlation and advanced Statistics classes
Instructional Examples/Lessons/Tasks	• Students can use everyday language to determine if two events are dependent. **Example:** Felix is a good chess player and a good math student. Do you think that the events "being good at playing chess" and "being a good math student" are independent or dependent? Justify your answer. **Example:** Juanita flipped a coin 10 times and got the following results: T, H, T, T, H, H, H, H, H, H. Her math partner Harold thinks that the next flip is going to result in tails because there have been so many heads in a row. Do you agree? Explain why or why not. • Students can explain conditional probability using everyday language. **Example:** A family that is known to have two children is selected at random from amongst all families with two children. Josh said that the probability of having two boys is 1/3 . Do you agree with Josh? Why or why not? Explain how you arrived at your answer? • Representing Conditional Probabilities (Mathematics Assessment Project)
Assessment Examples	• Illustrative Math Task 950 • Illustrative Math Task 951

Conditional Probability and the Rules of Probability

Cluster

Use the rules of probability to compute probabilities of compound events.

NVACS HSS.CP.B.6 (Major Supporting Work)

Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

Element	Exemplars
Standards for Mathematical Practice	• MP 8: Students do the same type of calculation several times and observe that □(□ □□□ □) = (□|□) · □(□)
Instructional Strategies	• Use hands on examples including, but not limited to: deck of cards, dice, and coins. • Connect the conditional probability to two-way tables by looking at specific columns/rows versus the total columns/rows
Prerequisite Skills	• Calculate probabilities using information represented in tables and Venn diagrams.
Connections Within and Beyond High School	• Use probability to investigate dependence and independence • Create and use two-way tables to approximate conditional probabilities
Instructional Examples/Lessons/Tasks	• Illustrative Mathematics o Unit 8 Lesson 8 Conditional Probability o Unit 8 Lesson 8 Practice Problems • RPDP Unit 9 - CP.B.6 • Khan Academy Conditional Probability Practice Problems
Assessment Examples	• Breadcrumbs activity

Conditional Probability and the Rules of Probability

Cluster

Use the rules of probability to compute probabilities of compound events.

NVACS HSS.CP.B.7 (Major Supporting Work)

Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Use the Addition Rule to get an answer and then interpret the meaning of their answer in a context. ● MP 3: Fix or find the error and explain correct reasoning.
Instructional Strategies	● Explain to students that P(A and B) needs to be subtracted from P(A or B) so that data is not counted twice. Explore a real simulation to illustrate this.
Prerequisite Skills	● Calculate probabilities using information represented in tables and Venn diagrams.
Connections Within and Beyond High School	● Within ○ Calculate probabilities using information represented in tables and Venn diagrams. ○ Use probability to determine whether or not two events are independent.
Instructional Examples/Lessons/Tasks	● Illustrative Mathematics: Geometry, Unit 8 Lesson 6 The Addition Rule (Illustrative Mathematics) ● Conditional Rules of Probability Unit 9: Probability S.CP.B.7 (RPDP)
Assessment Examples	● Geometry Unit 8, Lesson 6 Practice Problems (Illustrative Mathematics)

Conditional Probability and the Rules of Probability

Cluster

Use the rules of probability to compute probabilities of compound events.

NVACS HSS.CP.B.8 (Major Supporting Work)

(+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

Element	Exemplars
Standards for Mathematical Practice	• MP 8: Students do the same type of calculation several times and observe that □(□ □□□ □) = (□|□) · □(□) for events A and B.
Instructional Strategies	• Explain that $P(A \text{ and } B) = P(A\|B) \cdot P(B)$ can be re-written as $(\square\|\square) = \frac{\square(\square\ \square\square\square\ \square)}{\square(\square)}$.
Prerequisite Skills	• Calculate probabilities using information represented in tables and Venn diagrams. • Calculate Conditional probabilities using information represented in tables and Venn diagrams.
Connections Within and Beyond High School	• Investigate conditional probability and independence. • To recognize and practice finding probabilities for independent and dependent events.

Instructional Examples/Lessons/Tasks	• Geometry Unit 8
	  ○ Lesson 8 Conditional Probability (Illustrative Mathematics)
	  ○ Lesson 9 Using Tables for Conditional Probability (Illustrative Mathematics)
	• Multiplication Rule in Probability Example Problems (Varsity Tutors)

Assessment Examples	• Probability Review (Curriculum Engine)

Conditional Probability and the Rules of Probability

Cluster

Use the rules of probability to compute probabilities of compound events.

NVACS HSS.CP.B.9 (Major Supporting Work)

(+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Element	Exemplars
Standards for Mathematical Practice	• MP1: Students will understand why a given situation would be a permutation or a combination
Instructional Strategies	• Real World: How many different 6 digit number pin codes can be created? How does that change when codes include letters? Line up for graduation?
Prerequisite Skills	• Fraction Multiplication
Connections Within and Beyond High School	• Within: Fundamental Counting Principle • Beyond: Binomial Distribution
Instructional Examples/Lessons/Tasks	• Permutaions Random Walk III Random Walk IV Alex, Mel & Chelsea Play a Game Return to Fred's Fun Factory (Illustrative Mathematics) • Exploring Probability Too Many Choices How Likely Is it? Exploring Probability (Texas Instruments) • Permuations and Combinations PreCalc Mod 5 Topic A Lesson 4 PreCalc Mod 5 Topic A Lesson 2 (Engage NY) • Compound Events Probability of Compound Events (ACT Academy) • Probability using Permutations and Combinations Examples (Lumen) Probability with Permutations & Combinations (Khan Academy**)**
Assessment Examples	• Khan Assessment Practice Khan Academy Practice

Using Probability to Make Decisions

Cluster

Calculate expected values and use them to solve problems

NVACS HSS.MD.A.1 (Major Supporting Work)

(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

Element	Exemplars
Standards for Mathematical Practice	• MP 1: Students will summarize a situation with various outcomes by defining it as a random variable. • MP 5: Students will use various statistical techniques and graphical technologies to represent the random variable.
Instructional Strategies	• This is an advanced standard (+) and should be reserved for a statistics course or other equally advanced math course • Have students collect data of interest and form statistical questions of particular curiosity. • Make connections between data distributions (frequency) and probability distributions (relative frequency).
Prerequisite Skills	• Difference in data types (numerical vs. categorical) • Histograms
Connections Within and Beyond High School	• Correctly defining the problem is essential to perform the rest of the statistical work found in this cluster • This connection between probability and statistics is used in various social sciences.
Instructional Examples/Lessons/Tasks	• Probability Distribution (Khan Academy)
Assessment Examples	• AP Statistics Unit Resources (RPDP)

Using Probability to Make Decisions

Cluster

Calculate expected values and use them to solve problems

NVACS HSS.MD.A.2 (Major Supporting Work)

(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

Element	Exemplars
Standards for Mathematical Practice	• MP 4: The expected value is connected to a real world outcome and models possible results • MP 6: Students need to consider the context of their situations to determine the level of precision required
Instructional Strategies	• This is an advanced standard (+) and should be reserved for a statistics course or other equally advanced math course • Connect the multiplication of the probability and its related value to the mean • Use two-way table to examine a limited set of data to show the relationship
Prerequisite Skills	• Understand and be able to calculate a mean • Read and interpret tables of data
Connections Within and Beyond High School	• Students need to be able to find an expected value before they can find a complete distribution • This connection between probability and statistics is used in various social sciences.
Instructional Examples/Lessons/Tasks	• If you were to roll two die and sum their results multiple times, what would be the average sum? What is the expected value of the sum of two dice? Students can create a simulation, plot the results, and find the mean of the distribution. Then, they calculate the actual expected value using theoretical probabilities.
Assessment Examples	• AP Statistics Unit Resources (RPDP)

Using Probability to Make Decisions

Cluster

Calculate expected values and use them to solve problems

NVACS HSS.MD.A.3 (Major Supporting Work)

(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4: The probability distribution is connected to a real world outcomes and models the theoretical likelihood of possible results • MP 6: Students need to consider the context of their situations to determine the level of precision required
Instructional Strategies	• This is an advanced standard (+) and should be reserved for a statistics course or other equally advanced math course • This standard focuses on theoretical results, the next standard looked at empirical data • Give students a scenario like rolling three dice and counting the number of heads that are rolled. Have students determine all possible outcomes and the probability associated with each and display their results in a distribution table.
Prerequisite Skills	• Use of fundamental counting principle, permutations & combinations to calculate sample spaces. • Empirical probabilities vs. theoretical probabilities
Connections Within and Beyond High School	• Students need to be able to work with both theoretical and empirical situations and identify why each is appropriate in a given situation • This connection between probability and statistics is used in various social sciences.
Instructional Examples/Lessons/Tasks	• Expected Profit from a Lottery Ticket (Khan Academy)
Assessment Examples	• AP Statistics Unit Resources - (RPDP)

Using Probability to Make Decisions

Cluster

Calculate expected values and use them to solve problems

NVACS HSS.MD.A.4 (Major Supporting Work)

(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Element	Exemplars
Standards for Mathematical Practice	• MP 6: Students need to consider the context of their situations to determine the level of precision required
Instructional Strategies	• This is an advanced standard (+) and should be reserved for a statistics course or other equally advanced math course • This standard focuses on empirical data, the previous standard looked at theoretical results • Give students a scenario that provides experimental data. Have students determine all possible outcomes and the probability associated with each and display their results in a distribution table.
Prerequisite Skills	• Empirical probabilities vs. theoretical probabilities • Calculating simple probabilities
Connections Within and Beyond High School	• Students need to be able to work with both theoretical and empirical situations and identify why each is appropriate in a given situation • This connection between probability and statistics is used in various social sciences.
Instructional Examples/Lessons/Tasks	• Insurance with Expected Value (Khan Academy)
Assessment Examples	• AP Statistics Unit Resources– (RPDP)

Using Probability to Make Decisions

Cluster

Use probability to evaluate outcomes of decisions

NVACS HSS.MD.B.5 (Major Supporting Work)

(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

NVACS HSS.MD.B.5A (Major Supporting Work)

Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Students use worthwhile tasks and models that develop students' reasoning and making sense of probability and expected value ● MP 2: Students will use quantitative reasoning to compare and contrast different strategies and their associated probabilities and expected values. ● MP 3: Students will use different strategies to ensure reasonableness of payoff or winning values. ● MP 6: Students will attend to precision to determine exact expected payoff. ● MP 8: Students use repeated reasoning to justify and make sense of expected values for games and other scenarios over multiple iterations.
Instructional Strategies	● Heads-Up, manipulatives (dice, cards, spinners)
Prerequisite Skills	● Calculating expected values ● Chance investigation ● Evaluating probability models ● Tree diagrams ● Two-way tables
Connections Within and Beyond High School	● Expected value is used in many economic fields

Element	Exemplars
Instructional Examples/Lessons/Tasks	• Probability to Evaluate Fred's Fun Factory Sounds Really Good! (sort of…) (Illustrative Math) • Normal Distributions Candy Pieces Geometric Distributions Is It Rare? It's To Be Expected (Texas Instruments) • Expected Values Expected Values Making decisions with expected values (Khan Academy) • Games of Chance Precalc & Adv Topics Lesson 13 Precalc & Adv Topics Lesson 14 Precalc & Adv Topics Lesson 15 (Engage NY)
Assessment Examples	• Analyzing the Powerball Lottery Analyzing the Powerball Lottery (BetterLesson)

Using Probability to Make Decisions

Cluster

Use probability to evaluate outcomes of decisions

NVACS HSS.MD.B.5 (Major Supporting Work)

(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

NVACS HSS.MD.B.5B (Major Supporting Work)

Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

Element	Exemplars
Standards for Mathematical Practice	• MP 4: Students will model expected value problems based on real world situations.
Instructional Strategies	• Modify the game of PIG with dice for 2 dice
Prerequisite Skills	• Basic operations with fractions
Connections Within and Beyond High School	• This topic connects to many areas of consumer math and economics.
Instructional Examples/Lessons/Tasks	• Real world examples and an expected value calculator are found at Taylor Pearson
Assessment Examples	• Making Fair Decisions (Khan Academy)

Using Probability to Make Decisions

Cluster

Use probability to evaluate outcomes of decisions

NVACS HSS.MD.B.6 (Major Supporting Work)

(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Students will determine if a given situation is fair or not.
Instructional Strategies	• Create some simple simulations that students can experience the process of random decision-making.
Prerequisite Skills	• Simplifying and multiplying fractions. • Basic probability concepts for a single event.
Connections Within and Beyond High School	• The random decision making process has connections to real world situations in economics.
Instructional Examples/Lessons/Tasks	• Math Worksheet Island
Assessment Examples	• Making Fair Decisions (Khan Academy)

Using Probability to Make Decisions

Cluster

Use probability to evaluate outcomes of decisions

NVACS HSS.MD.B.7 (Major Supporting Work)

(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Element	Exemplars
Standards for Mathematical Practice	• MP 4: Students will model using probability in decision making for real world situations.
Instructional Strategies	• Open with a discussion of the Movie "Money Ball" and how the Oakland As used probability concepts to create a winning baseball team and how this process has expanded to all sports
Prerequisite Skills	• Students need knowledge of basic probability including both independent and dependent multiple event.
Connections Within and Beyond High School	• Review basic probability • Used in economic courses
Instructional Examples/Lessons/Tasks	• Have students do the Monty Hall simulation to show the complexity of decision making with probability. • Probability and the Monty Hall Problem (Khan Academy)
Assessment Examples	• Practice Test Problems 36 and 37 (RPDP)

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Southern RPDP

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Clark County School District

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Clark County School District

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Clark County School District

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Southern RPDP

Carly Strauss
Douglas County School District

Sara Swanson
Clark County School District

Trent Tietje
Douglas County School District

Lynn Trell
Clark County School District

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Clark County School District

Wendy Weatherwax
Clark County School District

References

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Jhone Ebert

Superintendent of Public Instruction

Jonathan Moore, Ed.D

Deputy Superintendent for Student Achievement

Dave Brancamp

Director of Standards and Instructional Support

Tracy Gruber

K-12 Mathematics Education Program Professional