Mathematics Standards Clarification for Number & Quantity Conceptual Category

High School

The Nevada Ready! Network

Designed for teachers by teachers!

2019

The Nevada Ready! Network

Standards-Based Instruction for ALL Nevada Students

Nevada Department of Education

Nevada Ready!

Contents

The Real Number System

Cluster:

Extend the properties of exponents to rational exponents.

NVACS HSN.RN.A.1

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

Element	Exemplars
Standards for Mathematical Practice	• MP 1: Students will make sense of problems and persevere in solving problems by rewriting exponential and rational expressions. • MP 7: Students will look for and make use of structure by applying properties of exponents and radicals to simplify expressions.
Instructional Strategies	• Use problems such as $(\sqrt{3})^2$ and $\sqrt{((3^2))}$ to consider exponents. • Provides problems that give a context for using rational exponents. • Presses students to explain the meaning of a fractional in different contexts. • Provides problems that enable students to determine the difference between computing a value such as $\sqrt{4}$ and finding the solution set of a related equation, such as $x^2=4$.
Prerequisite Skills	• Know perfect squares and cubes. • Know properties of exponents. • Know the definition of a rational exponent.
Connections Within and Beyond High School	• Properties of exponents. • Graphing radical equations. • Simplifying radicals. • Solving equations using radicals and logarithms.
Instructional Examples/Lessons/Tasks	• Achieve the Core (Achieve the Core) • MVP Math Lesson 2.4, 2.6, 2.7 (MVP Math) • Illustrative Math 3 Tasks (Illustrative Math)
Assessment Examples	• Big Ideas Math Chapter 5 Test page 289, exercises 5–8 (Big Ideas)

The Real Number System

Cluster:

Extend the properties of exponents to rational exponents.

NVACS HSN.RN.A.2

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Element	Exemplars
Standards for Mathematical Practice	• MP 1: Students will make sense of problems and persevere in solving problems by rewriting radicals as expressions with rational exponents. • MP 7: Students will look for and make use of structure by applying properties of exponents and radicals to simplify expressions.
Instructional Strategies	• Use problems that allow students to use either radical or exponential forms and requires them to explain their reasoning for their choice.
Prerequisite Skills	• Know perfect squares and cubes. • Know properties of exponents. • Know the definition of a rational exponent.
Connections Within and Beyond High School	• Properties of exponents. • Graphing radical equations. • Simplifying radicals. • Solving equations using radicals and logarithms.
Instructional Examples/Lessons/Tasks	• Achieve the Core (Achieve the Core) • MVP Math Lesson 2.4, 2.6 (MVP Math)
Assessment Examples	• Shmoop Click Sample Assessments for Drills (Schmoop)

The Real Number System

Cluster:

Use properties of rational and irrational numbers.

NVACS HSN.RN.B.3

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Element	Exemplars
Standards for Mathematical Practice	● MP 3: Students will construct viable arguments and critique the reasoning of others as they use results about the sums and products of rational and irrational numbers to determine whether a sum or product is rational, and to justify their reasoning.
Instructional Strategies	● Have students explore computations with only rationals to get a feel for what it means for a computation to have a value that is or is not part of the set being used. ● Have students explore different computations with irrationals. ● Have students explore different computations with rational and irrational numbers. ● Have students explain and justify their answers.
Prerequisite Skills	● Classify numbers as rational or irrational. ● Know and apply the properties of equality.
Connections Within and Beyond High School	● Closure property.
Instructional Examples/Lessons/Tasks	● Achieve the Core (Achieve the Core)
Assessment Examples	● Shmoop Click Sample Assessments for Drills (Schmoop)

The Real Number System

Cluster:

Reason quantitatively and use units to solve problems.

NVACS HSN.Q.A.1 (Major Supporting Work)

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Make sense of the quantities and their relationship within the context of the problem. ● MP 4: Model the quantities within the context of the problem to apply to solve everyday situations ● MP 7: Look for repeated reasoning in calculations for shortcuts. (For example, some students will convert yards straight to inches instead of converting yards to feet and then feet to inches.)
Instructional Strategies	● This standard also includes converting measurements and dimensional analysis. ● Using, choosing, and interpreting units should occur in the context of applications which contain them, especially mathematical modeling and formulas. For example: When applying the formula *distance = (speed)(time)*, students should recognize that when distance is measured in km, and speed is measured in km/hr, then time must be measured in hours. ● Attention should be made to the scale of each axis from the origin of graphs in all graphing situations, especially those where data and application are present. This includes choosing appropriate viewing windows on graphing calculators and software. For example: When graphing an exponential function, the horizontal scale may be in ones but the vertical scale in tens, hundreds, thousands, or more. Also, the graph may not need to show negative values of the dependent variable, depending on the context or function. For example: When fitting a line to data, a graph may not include the origin to best display the data.
Prerequisite Skills	● Understand reciprocals. ● Understand unit rates, rate of change, and proportions. ● Unit conversions of measurement. ● Interpreting graphs. ● Breaking down modeling problems.

Element	Exemplars
Connections Within and Beyond High School	• Link to their science courses (The science teachers can be very helpful with this topic.) • Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions. • Slope and average rate of change. • Modeling.
Instructional Examples/Lessons/Tasks	• Blast Module (Blast) • Illustrative Mathematics 10 Tasks (Illustrative Math)
Assessment Examples	• RPDP Math, High School, Unit 2 Relationships Between Quantities Notes, Pages 9–11 (RPDP)

Quantities

Cluster:

Reason quantitatively and use units to solve problems.

NVACS HSN.Q.A.2 (Major Supporting Work)

Define appropriate quantities for the purpose of descriptive modeling.

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Make sense and explain the meaning of the problem, as well as the quantities and their relationships in the problem. ● MP 3: Attend to precision when reporting solutions to problems requiring discrete responses.
Instructional Strategies	● Mathematical models require specifically defined inputs and outputs, measured in appropriate units. There should be attention to this whenever applications are employed, whether linear, quadratic, exponential, or other type of function in this course. For example: Let T = total cost of a taxi ride in dollars, let d = distance of the taxi in miles, where each 1/7 mile costs 15 cents, and let f = the flat fee for a taxi ride in dollars. A model for this would be T = f + (0.15)(7d). ● Create discourse with students to check that an answer is reasonable.
Prerequisite Skills	● Understand reciprocals. ● Understand unit rates, rate of change, and proportions. ● Unit conversions of measurement. ● Interpreting graphs. ● Breaking down modeling problems.
Connections Within and Beyond High School	● Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions. ● Modeling.
Instructional Examples/Lessons/Tasks	● Blast Module (Blast) ● Shmoop Click Samples Assessment for Drills (Schmoop) ● Spark 101 NASA Bone Density in Space (Spark)
Assessment Examples	● Illustrative Mathematics 3 Tasks (Illustrative Math)

Quantities

Cluster:

Reason quantitatively and use units to solve problems.

NVACS HSN.Q.A.3 (Major Supporting Work)

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Element	Exemplars
Standards for Mathematical Practice	● MP 1: Make sense of the meaning of the problem in context. ● MP 3: Attend to precision by applying rules of significant digits. ● MP 7: Look for repeated reasoning in calculations for shortcuts. (For example, some students will convert yards straight to inches instead of converting yards to feet and then feet to inches.)
Instructional Strategies	● Intermediate calculations should not be rounded; rounding to the appropriate degree of precision occurs only at the end of a string of computations. ● Solutions should be addressed both from a conceptual / common sense level and from a formal perspective. For example: Conceptually, if the sides of a rectangle are measured to tenths of centimeters, then the calculated area should not be reported to a greater precision than tenths of square centimeters. For example: Formally, if the sides of a rectangle are measured such that the lengths have two and three significant digits respectively, then the area should be reported to two significant digits.
Prerequisite Skills	● Rounding rules
Connections Within and Beyond High School	● Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions. ● Link to their science courses (The science teachers can be very helpful with this topic). ● Checking for a reasonable solution. ● Modeling.
Instructional Examples/Lessons/Tasks	● Blast Module (Blast) ● Illustrative Mathematics 7 Tasks (Illustrative Math) ● Math is Fun Help to clarify accuracy versus precision (MathisFun) ● MathBits Notebook Working with units (MathBits)
Assessment Examples	● RPDP Math, High School, Unit 2 Relationships Between Quantities Notes, Page 6 (RPDP)

The Complex Number System

Cluster:

Perform arithmetic operations with complex numbers.

NVACS HSN.CN.A.1 (Major Supporting Work)

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will reason abstractly to identify imaginary numbers. • MP 3: Construct viable arguments and critique the reasoning of others. Example: Defend or analyze the work of others.
Instructional Strategies	• Finding patterns for the powers of *i* using multiplication, rules of exponents, and the definition of *i*. • Identifying and categorizing real and complex numbers.
Prerequisite Skills	• Definition of like terms. • Definition of a coefficient. • Knowledge of number sets.
Connections Within and Beyond High School	• Solving Quadratic Equations. • Restrictions on the Domain on Radical Functions. • Polar and Cartesian Forms of Complex Numbers. • Modular Arithmetic.
Instructional Examples/Lessons/Tasks	• Illustrative Math Complex Numbers and Patterns (Powers of *i*) (Illustrative Math) • RPDP Identify the discriminant of a quadratic and describe the roots (RPDP)
Assessment Examples	• RPDP Unit Test (RPDP)

The Complex Number System

Cluster:

Perform arithmetic operations with complex numbers.

NVACS HSN.CN.A.2 (Major Supporting Work)

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Students will use abstract reasoning to understand the value of squaring an imaginary number. ● MP 8: Students will develop an understanding of what happens when operations are performed on imaginary numbers through the use of repeated reasoning.
Instructional Strategies	● Connect to operations on scientific notation as a review of properties. ● Connect to operations on binomials as a review of properties.
Prerequisite Skills	● Powers of *i*. ● Distributive Property, especially for two binomials. ● Commutative Property for regrouping like terms. ● Associative Property.
Connections Within and Beyond Grade Level	● Solving quadratic equations. ● Graphing on the Complex plane. ○ Rectangles, Squares, Cubes ● Vectors and Polar Graphs
Instructional Examples/Lessons/Tasks	● Complex Number Products Greatest Value (Open Middle) ● Complex Number Products (Open Middle) ● Multiply Complex Numbers (Open Middle) ● Factoring Complex Numbers (Open Middle) ● Engage NY Lesson 37 (EngageNY)
Assessment Examples	● RPDP Unit Test (RPDP)

The Complex Number System

Cluster:

Perform arithmetic operations with complex numbers.

NVACS HSN.CN.A.3 (Major Supporting Work)

Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will use abstract reasoning to understand the value of squaring and imaginary number. • MP 8: Students will develop an understanding of what happens when operations are performed on imaginary numbers through the use of repeated reasoning.

Instructional Strategies	• Open with $(5+2\square)(\square+\square\square)$ have students determine values of *a* and *b* using the numbers between –9 and 9 such that the product is a real number.
	  ○ Give each table group a different example eg: $(6+4\square)(\square+\square\square)$ and have each group determine which values of *a* and *b* that yield a real product.
	  ○ Students then share out their results and identify the pattern to determine the conjugate.

Prerequisite Skills	• Define moduli as the plural of modulus, which is the absolute value of a complex number. • Operations on complex numbers. • Distance formula. • Pythagorean Theorem.
Connections Within and Beyond High School	• Rationalizing a denominator.
Instructional Examples/Lessons/Tasks	• pp 18–21 (RPDP) • Complex Numbers Worksheet (RPDP) • CPalms
Assessment Examples	• RPDP pp 21–22 (RPDP) • Shmoop Click Samples Assessments for Drills (Schmoop)

The Complex Number System

Cluster:

Represent complex numbers and their operations on the complex plane.

NVACS HSN.CN.B.4 (Major Supporting Work)

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Element	Exemplars
Standards for Mathematical Practice	● MP 2: Students will use abstract reasoning to understand the value of squaring and imaginary number. ● MP 8: Students will develop an understanding of what happens when operations are performed on imaginary numbers through the use of repeated reasoning.
Instructional Strategies	● Use scaffolding to build the concept as follows: ○ A single real number on the rectangular plane to the polar plane. ○ An imaginary unit on the rectangular plane to the polar plane. ○ Finally, a complex number on the rectangular plane to the polar plane.
Prerequisite Skills	● Graphing on a Cartesian Plane. ● Distance formula and Pythagorean Theorem. ● Right triangle trigonometry. ● Degree to radian conversions.
Connections Within and Beyond High School	● Transformations. (Dilations & Rotations)
Instructional Examples/Lessons/Tasks	● Engage NY Lesson 13 (Engage NY) ● CPalms Virtual Manipulative and Video (cpalms) ● TI Calculator Activities Texas Instruments)
Assessment Examples	● Exit Ticket Lesson 13 (Engage NY)

The Complex Number System

Cluster:

Represent complex numbers and their operations on the complex plane.

NVACS HSN.CN.B.5 (Major Supporting Work)

(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120.*

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will use abstract reasoning to understand the value of squaring an imaginary number. • MP 8: Students will develop an understanding of what happens when operations are performed on imaginary numbers through the use of repeated reasoning.
Instructional Strategies	• Introduce using powers of *i* graphed on the complex plane. • Draw a vector diagram of each complex number to model operations geometrically.
Prerequisite Skills	• Operations with like terms, binomials, and complex numbers. • Pythagorean Theorem. • Right triangle trigonometry. • Transformations.
Connections Within and Beyond High School	• Graphing complex numbers on the complex plane. • Vector operations.
Instructional Examples/Lessons/Tasks	• Geometric representation of Complex Number operations using Vectors (Math Facutly) • TI Calculator Activities (Texas Instruments) • Addition & Subtraction (Engage NY) • Multiplication (Engage NY) • Illustrative Math 1 Task (Illustrative Math)
Assessment Examples	• Exit Ticket pp 7–9 (Engage NY) • Exit Ticket p 121 (Engage NY)

The Complex Number System

Cluster:

Represent complex numbers and their operations on the complex plane.

NVACS HSN.CN.B.6 (Major Supporting Work)

(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will use abstract reasoning to understand the value of squaring and imaginary number. • MP 8: Students will develop an understanding of what happens when operations are performed on imaginary numbers through the use of repeated reasoning.
Instructional Strategies	• Help the students to understand this complex topic by relating it to finding the distance between any two points in the Cartesian Plane.
Prerequisite Skills	• Graphing in the Cartesian Plane. • Distance Formula. • Pythagorean Theorem.
Connections Within and Beyond High School	• Finding the distance between points. • Connects to work that will be done with vectors.
Instructional Examples/Lessons/Tasks	• Modulus of the Difference & Midpoint (Engage NY) • Midpoint of a Segment (Engage NY) • RPDP Notes pp 20–22 (RPDP) • Illustrative Math Complex Distance (Illustrative Math)
Assessment Examples	• Exit Ticket pp 7–10 (Engage NY)

The Complex Number System

Cluster:

Use complex numbers in polynomial identities and equations.

NVACS HSN.CN.C.7 (Major Supporting Work)

Solve quadratic equations with real coefficients that have complex solutions.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will use abstract reasoning to understand how a problem with real numbers can generate a solution that includes imaginary numbers. • MP 6: Students will need to be precise in dealing with negative quantities as these will be source for their complex solutions.
Instructional Strategies	• Graph the parent function $y=x^2$. Ask what are the solutions to this graph? What would be the equation if we shift this function 4 units down? What are the solutions to this graph? What if we shift the parent function up 4 units? What is the equation? What are its solutions?
Prerequisite Skills	• Quadratic formula • Simplifying radicals • Completing the square (including area modeling)
Connections Within and Beyond High School	• Solving quadratic equations. • Solving polynomial equations.
Instructional Examples/Lessons/Tasks	• Illustrative Mathematics 1 Task (Illustrative Math) • Engage NY Lesson 4 (Engage NY) • Interactive Maths: Solving Quadratic Equations (select Complex option) (Interactive Maths) • RPDP pp 21–30 (RPDP)
Assessment Examples	• Engage NY Exit Ticket p 54 (Engage NY) • RPDP Multiple Choice Questions pp 13–14, 29-30 (RPDP)

The Complex Number System

Cluster:

Use complex numbers in polynomial identities and equations.

NVACS HSN.CN.C.8 (Major Supporting Work)

(+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.

Element	Exemplars
Standards for Mathematical Practice	• MP 7: Students will use the structural similarity between polynomial expressions and complex numbers to apply the rules for operations from the polynomials to the complex.
Instructional Strategies	• Start with a review of factoring polynomials with real solutions, then extend it to the complex numbers.
Prerequisite Skills	• Factoring polynomials. • Polynomial operations.
Connections Within and Beyond High School	• Factoring Polynomials • Properties of Real Numbers such as Associative, Commutative, Distributive & Identity Properties.
Instructional Examples/Lessons/Tasks	• Illustrative Mathematics 1 Task (Illustrative Math) • Engage NY Lesson 3 (Engage NY)
Assessment Examples	• Exit Ticket pp 11 (Engage NY)

The Complex Number System

Cluster:

Use complex numbers in polynomial identities and equations.

NVACS HSN.CN.C.9 (Major Supporting Work)

(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Element	Exemplars
Standards for Mathematical Practice	• MP 8: Look for and express regularity in repeated reasoning to discover the nature of the Fundamental Theorem of Algebra.
Instructional Strategies	• Connect the roots of a quadratic graph to the solutions of an equation to the linear factors of the equation. Then extend to complex solutions and linear factors. Then extend to polynomial functions. • Create discourse among students regarding the use of the words roots, zeros, and solutions. These words, although not interchangeable, have very similar meanings.
Prerequisite Skills	• Definition of linear factors. • Degree of a polynomial. • Remainder Theorem. • Repeated Zeros. • End behavior of graphs.
Connections Within and Beyond High School	• *x*-intercepts of Polynomial Functions • Odd/Even Roots
Instructional Examples/Lessons/Tasks	• Engage NY Lesson 40 (Engage NY) • TI Calculator Activities (Texas Instruments)
Assessment Examples	• Exit Ticket p 477 (Engage NY)

Vector and Matrix Quantities

Cluster:

Represent and model with vector quantities.

NVACS HSN.VM.A.1 (Major Supporting Work)

(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).

Element	Exemplars
Standards for Mathematical Practice	• MP2: When students understand that vectors are quantities they extend their ability to use quantitative reasoning.
Instructional Strategies	• Introduce vectors by looking at them in a real world context. For example, discuss the difference between velocity (vector) and speed (magnitude). • Relate vector magnitude to the distance formula. • Take time to expose students to the notations used to describe vectors.
Prerequisite Skills	• Pythagorean Theorem.
Connections Within and Beyond High School	• Connect vectors to polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp 1–5 (RPDP) • Collaborate with science instructors (physics) for examples of projects utilizing vectors.
Assessment Examples	• RPDP (RPDP)

Vector and Matrix Quantities

Cluster:

Represent and model with vector quantities.

NVACS HSN.VM.A.2 (Major Supporting Work)

(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

Element	Exemplars
Standards for Mathematical Practice	• MP 1: Students will make sense of vector problems by looking at the components of the vectors as coordinate changes.
Instructional Strategies	• Create right triangles on the coordinate plane where the given vector is the hypotenuse and the legs become the components of the vectors. • Move vectors to different locations and with different directions to explore how these relate to the components of the vector. • Treat the starting point of a vector as a translation from the origin.
Prerequisite Skills	• Coordinate geometry. • Pythagorean Theorem.
Connections Within and Beyond High School	• Connect vectors to polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp 1–5 (RPD) • TI Calculator Activity (Texas Instruments) • Collaborate with science instructors (physics) for examples of projects utilizing vectors.
Assessment Examples	• RPDP (RPDP)

Vector and Matrix Quantities

Cluster:

Represent and model with vector quantities.

NVACS HSN.VM.A.3 (Major Supporting Work)

(+) Solve problems involving velocity and other quantities that can be represented by vectors.

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Students will justify their representations of vector quantities and critique the representations of other students. • MP 4: Students will be able to use vectors to model real world situations.
Instructional Strategies	• Provide examples that relate to force, momentum, velocity, and work. • Connect with the Physics teacher for some examples that will build upon the work students do in that class.
Prerequisite Skills	• The concepts of units and rates. • Right triangle trigonometry.
Connections Within and Beyond High School	• Vectors are used if you study polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp 6–9 (RPDP) • Vectors & Sports (Prezi)
Assessment Examples	• RPDP (RPDP) • Shmoop Click sample assessments for drills. (Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on vectors.

NVACS HSN.VM.B.4 (Major Supporting Work)

(+) Add and subtract vectors.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students reason abstractly and quantitatively by adding and subtracting vectors algebraically and graphically, using different representations to find solutions.
Instructional Strategies	• Add and subtract vectors algebraically and graphically.
Prerequisite Skills	• Simplifying expressions by combining like terms. • Basic coordinate geometry, plotting points.
Connections Within and Beyond High School	• Scalar multiplication and vector multiplication. • Sum of magnitude/direction. • Connections to right triangle trigonometry. • Polar and complex numbers. • Geometric representation of vector operations using parallelograms on the coordinate plane.
Instructional Examples/Lessons/Tasks	• Graphing Adding and Subtracting Vectors (Khan Academy) • Sum of Two Vectors (Wolfram Demonstrations Project) • Open Middle (Open Middle) • Shmoop Click sample assignments for drills (Schmoop)
Assessment Examples	• Algebraically Add and Subtract Vectors Practice (Khan Academy) • Graphically Add and Subtract Vectors Practice (Khan Academy) • RPDP (RPDP)

Vector and Matrix Quantities

Cluster:

Perform operations on vectors.Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

NVACS HSN.VM.B.4.A (Major Supporting Work)

Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: The parallelogram rule allows students to use quantitative reasoning of vector components to add vectors. • MP 4: Students will model vector addition using the parallelogram rule.
Instructional Strategies	• Placing the vectors on a coordinate grid allows students to see how the components of the original vectors relate to the resulting addition. • Help students to understand how the parallelogram rule relate to basic geometry work students have done in the past. • Parallelogram rule is $\sqrt{V_1^2 + V_1^2 + V_1 V_2 \cos(\textit{angle formed by the two vectors})}$
Prerequisite Skills	• Pythagorean Theorem. • Basic trigonometry.
Connections Within and Beyond High School	• Law of Cosines. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp 1–5 (RPDP) • TI Calculator Activity (Texas Instruments) • Engage NY Lesson 19 (Engage NY)
Assessment Examples	• RPDP (RPDP)

Vector and Matrix Quantities

Cluster:

Perform operations on vectors.

NVACS HSN.VM.B.4.B (Major Supporting Work)

Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

Element	Exemplars
Standards for Mathematical Practice	• MP 7: Using their knowledge of the structure of a vector students will be able to add them when given only a direction and magnitude of the original vectors.
Instructional Strategies	• Have students practice moving between different representations of vectors so that they can apply the formulas when needed. • Have students create both triangle representations and parallelogram representations to determine which they find more useful in different situations.
Prerequisite Skills	• Pythagorean Theorem. • Basic trigonometry.
Connections Within and Beyond High School	• Connect vectors to polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp1–5 (RPDP) • Engage NY Lesson 20 (Engage NY)
Assessment Examples	• RPDP (RPDP)

Vector and Matrix Quantities

Cluster:

Perform operations on vectors.

NVACS HSN.VM.B.4.C (Major Supporting Work)

Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

Element	Exemplars
Standards for Mathematical Practice	• MP 7: Using their knowledge of the structure of a transformation of functions students will be able to understand $a - v$ is equivalent to $a + (-v)$.
Instructional Strategies	• Remind students of the additive inverse as a way to understand subtracting vectors. • Use a coordinate plane to geometrically represent subtraction of vectors.
Prerequisite Skills	• Knowledge of vector components.
Connections Within and Beyond High School	• Vectors are used in the study of polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp1–5 (RPDP) • Engage NY Lesson 17 (Engage NY)
Assessment Examples	• RPDP (RPDP)

Vector and Matrix Quantities

Cluster:

Perform operations on vectors.

NVACS HSN.VM.B.5 (Major Supporting Work)

(+) Multiply a vector by a scalar.

NVACS HSN.VM.B.5.A (Major Supporting Work)

Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

Element	Exemplars
Standards for Mathematical Practice	• MP 8: By exploring repeated addition of vectors students will understand the effect of scalar multiplication on a vector.
Instructional Strategies	• Use repeated addition of vectors to show what scalar multiplication looks like. • Be certain to include adding repeated negative vectors so students will see what multiplying with a negative scalar does. • Relate scalar multiplication to dilations of points. • Represent contextual situations with the use of vectors and scalars.
Prerequisite Skills	• Students need to master vector addition before moving on to scalar multiplication.
Connections Within and Beyond High School	• Vectors are used in the study of polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp 6–9 (RPDP) • CPalms Linear Motion (Cpalms)
Assessment Examples	• RPDP (RPDP) • Shmoop Click sample assignment for drills. (Scnmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on vectors.

NVACS HSN.VM.B.5 (Major Supporting Work)

(+) Multiply a vector by a scalar.

NVACS HSN.VM.B.5.B (Major Supporting Work)

Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Students will apply their knowledge of scalar multiplication to reason out the effect of a scalar on the magnitude of a vector.
Instructional Strategies	• Have students find the resulting vector from a scalar multiplication. • Compare the magnitude of the original vector to the magnitude of the scalar and see if there is a pattern. • Be sure to include a negative scalar while doing these comparisons.
Prerequisite Skills	• Scalar multiplication of vectors.
Connections Within and Beyond High School	• Vectors are used in the study of polar coordinates. • Vectors are used in many higher-level science courses to describe and explain physical movements and forces.
Instructional Examples/Lessons/Tasks	• RPDP pp 6–9 (RPDP)
Assessment Examples	• RPDP (RPDP) • Shmoop Click sample assignments for drills. (Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.6 (Major Supporting Work)

(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

Element	Exemplars
Standards for Mathematical Practice	• MP 4: Use problem situations and texts to represent matrices in different ways. • MP 7: Connect representations of matrices to make connections of its structures and the context.
Instructional Strategies	• Represent data in a matrix and determine the dimensions of a matrix. • Require students to model contextual situations with matrices. • Allow students to explore connections between representations of matrices and to use matrix operations for specific purposes. • Provide opportunities for students to use matrix software to represent large data sets and perform operations on these data sets.
Prerequisite Skills	• Understand row vs column. • Add, subtract, multiply, divide rational numbers.
Connections Within and Beyond High School	• Organizing data in matrices leads to justification for specific operations on a data set. This will lead to exploration of similar relationships with frequency tables in the probability and statistics conceptual category.
Instructional Examples/Lessons/Tasks	• Better Lesson (Better Lesson) • Engage NY Lesson 1 (Engage NY)
Assessment Examples	• Shmoop Click on Sample Assignments then Drill. (Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.7 (Major Supporting Work)

(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

Element	Exemplars
Standards for Mathematical Practice	● MP 4: Represent contextual situation using matrices and manipulate the appropriate matrix using scalar multiplication.
Instructional Strategies	● Provide multiple contexts to use scalar multiplication with matrices. ● Provide opportunities to explore the relationship of scalar multiplication to solve different problems.
Prerequisite Skills	● Multiplication of rational numbers.
Connections Within and Beyond High School	● Dilating a figure that is represented by a matrix on the coordinate plane. ● Increasing the cost of merchandise at a store to fit a desired profit margin. ● Manipulating a data set to have a desired mean and/or standard deviation.
Instructional Examples/Lessons/Tasks	● Better Lesson (Better Lesson)
Assessment Examples	● Shmoop Click on Sample Assignments then Drill. (Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.8 (Major Supporting Work)

(+) Add, subtract, and multiply matrices of appropriate dimensions.

Element	Exemplars
Standards for Mathematical Practice	• MP 6: Attend to precision by adding and subtracting specific elements of different matrices. • MP 4: Model matrix operation situations from real world contexts.
Instructional Strategies	• Start with the addition and subtraction of matrices, and relate this to appropriate dimensions. • Use dimensions to verify matrix operations can be performed before performing them. • Allow students opportunities to explore matrix operations with technology.
Prerequisite Skills	• Add, subtract, and multiply with rational numbers.
Connections Within and Beyond High School	• Model transformations such as rotations with the use of matrix multiplication. • The use of one matrix addition or subtraction followed by a matrix multiplication can standardize an entire data set.
Instructional Examples/Lessons/Tasks	• Part 1 (Better Lesson) • Part 2 (Better Lesson) • Open Middle (Engage NY) • TI Calculator Activities (Texas Instruments)
Assessment Examples	• Schmoop Click on Sample Assignments then Drill. (Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.9 (Major Supporting Work)

(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

Element	Exemplars
Standards for Mathematical Practice	• MP 3: Construct viable arguments to justify why matrix operations have or do not have certain properties.
Instructional Strategies	• Allow students to explore the multiplication of different matrices in different orders to verify that the commutative property is not satisfied. • Justify and provided counter-examples for why matrix operations do not have certain properties. • Through exploration, students discover that some properties of real numbers will not exist for matrices.
Prerequisite Skills	• Distributive Property • Associative Property • Commutative Property
Connections Within and Beyond High School	• Provide connections of matrix properties to other contexts and standards. Specifically, relate these properties to other operational properties, such as the identity property. • Connect dimensional analysis of matrices to specific matrix operations.
Instructional Examples/Lessons/Tasks	• Better Lesson identify which real number properties can be used with matrices. (Better Lesson)
Assessment Examples	• Schmoop Click on Sample Assignments then Drills. (Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.10 (Major Supporting Work)

(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Element	Exemplars
Standards for Mathematical Practice	• MP 6: Attend to precision when applying properties of real numbers to matrices.
Instructional Strategies	• Allow students to explore and experience different examples of the identity property in varying dimensional sizes. • Use the identity property of matrices to develop an understanding of the inverse of a matrix. • Allow time for students to relate matrix multiplication to the identity matrix and how the procedure for multiplication produces the same matrix.
Prerequisite Skills	• Zero property • Identity property • Multiplicative inverse
Connections Within and Beyond High School	• A determinate gives quick information about the number of solutions to a system of equations. • For a square matrix, the determinate tells if an inverse exists.
Instructional Examples/Lessons/Tasks	• Better Lesson compute determinants and use them to find the inverse of a matrix. (Better Lesson) • Shmoop (Schmoop)
Assessment Examples	• Shmoop Click on Sample Assignments then Drill. (Schmoop) • TI Calculator Activities Questions and answers can be used with/without calculator. (Texas Instruments)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.11 (Major Supporting Work)

(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Element	Exemplars
Standards for Mathematical Practice	• MP 2: Reason abstractly and quantitatively to connect matrix multiplication to vector transformations.
Instructional Strategies	• Use graphical models on the coordinate plane to connect vector multiplication and with transformational matrices. • Model different dilations of polygons using matrix multiplication.
Prerequisite Skills	• Knowledge of dilations • Matrix multiplication
Connections Within and Beyond High School	• Multiplying a matrix and a vector is really just a combination of multiplication and addition or a way to transform that vector in it into two or three-dimensional space. This standard is an extension of vector and matrix standards earlier in this domain. When we multiply a matrix and a vector together, we transform that vector in a way that changes its components and representation in space.
Instructional Examples/Lessons/Tasks	• Khan Academy Transform vectors using matrices video. (Khan Academy)
Assessment Examples	• Khan Academy Transform vectors using matrices practice problems. (Khan Academy) • Khan Academy Matrices as transformations practice problems. (Khan Academy) • Shmoop Click sample assignments for drills.(Schmoop)

Vector and Matrix Quantities

Cluster:

Perform operations on matrices and use matrices in applications.

NVACS HSN.VM.C.12 (Major Supporting Work)

(+) Work with 2 × 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.

Element	Exemplars
Standards for Mathematical Practice	• MP7: Look for structure by seeing a transformation as vectors in matrix form. • MP 8: Use repeated reasoning to determine the relationship between scalar multiplication of the transformation matrix and area of the related figure.
Instructional Strategies	• Wolfram Demonstration Project shows sliders that you can manipulate to rotate objects. • Geometry of Linear Transformations of the Plane Rotation, Reflection, Expansion & Compression, Shears.
Prerequisite Skills	• Calculate the determinant of a square matrix.
Connections Within and Beyond High School	• Rotations of plane figures • Cramer's Rule
Instructional Examples/Lessons/Tasks	• Better Lesson: Warm-ups, discovery & application, worksheet, wrap-up (Better Lesson) • Open Ed (Acta Academy) • You Tube (You Tube) • Area of Polygons (Texas Instruments) • The Determinant of a Matrix (Texas Instruments)
Assessment Examples	• Shmoop Click on Sample Assignments then Drills. (Schmoop)

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References

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